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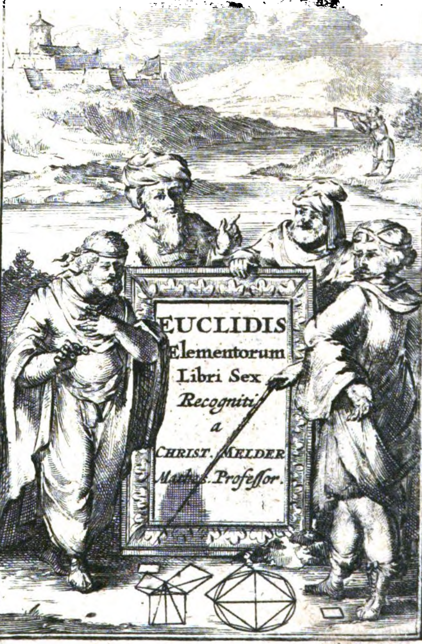
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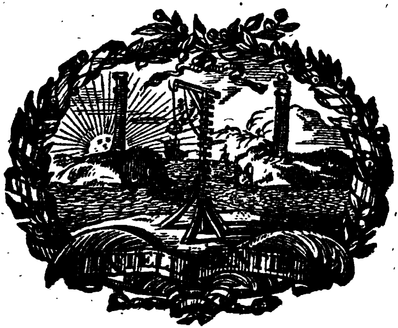
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PRÆFATIO

AD

LECTOREM.

Inter plurimos qui ſex priora Euclidis Elementa commentariis illuſtrarunt non minimam laudem meretur Georgius Fournier. Qui prolixas obſcuraſque demonſtrationes evitando, claras ac ſuccinctas ſubſtituit, Le-

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ctorum attentionem ſine imaginationis confuſione ut ſibi conciliaret.

Præter figurarum intricatam exiguitatem primum nil diſplicuit; quas proinde ſimpliciter mutare decreveram: Sed in ipſo operis proceſſu non tantum multa ex Clavio, Tacqueto, Barrow aliiſque adjeci, verum perplurimas demonſtrationes ita immutavi, præſertim in poſterioribus libris, ut nullo

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nullo modo nomen meum reticere potuerim; quod in hunc finem moneo, ne quis me injuriam D° Fournier feciſſe putet. Aliorum labores pro meis venditare nec ſtudeo nec ſoleo. Agnoſco pleraque ipſius eſſe. Correctiora vel ante annum prodiiſſent, niſi execrabilis bellorum turba, variaque hinc nata impedimenta interceſſiſſent. Cæterum applauſum ſi obtinuerint

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quæ appoſui ad meliora ac magis grata inſtigabor. Vale.

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EUCLIDIS ELEMENTUM PRIMUM.

DEFINITIONES.

1. *Punctum eſt, cujus pars nulla.*

Græcè legitur σημεῖον, ſignum hoc eſt à quo incipit deſignatio quantitatis finitæ. Idem intellige de linea ac ſuperficie, non quod ex fluxu puncti aut lineæ originem traxerint.

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2. *Linea vero longitudo non lata.*

Linea talis nulla ducitur à parte rei; ſed ſicut punctum, ita & linea ſignum ſeu initium eſt quantitatis latæ.

3. *Lineæ autem termini ſunt puncta.*

Id eſt longitudinis determinatæ principium & finis eſt punctum: per infinitam autem lineam Euclides intelligit lineam cujuſvis magnitudinis, ſeu indeterminatam.

4. *Recta linea eſt, quæ ex æquo ſua interjacet puncta.*

Sive cujus extrema obumbrant omnia media, ut dixit Plato: vel minima earum quæ terminos habent

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bent eoſdem, ut vult Archimedes.

5. *Superficies vero eſt quæ longitudinem latitudinemque tantum habet.*

6. *Superficiei autem extrema ſunt lineæ.*

Hæc definitio intelligenda eſt tantùm de ſuperficie plana vel mixta, non autem de circulari; quando enim habet extremum, lineam tantùm habet, non lineas.

7. *Plana ſuperficies, eſt quæ ex æquo ſuas interjacet rectas.*

Quæ dixi de linea recta, eadem de plana ſuperficie ſunt intelligenda.

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8. *Planus autem angulus eſt duarum linearum in plano ſe mutuo tangentium, & non in directum jacentium, alterius ad alteram inclinatio.*

Hic cauſæ anguli explicantur: Materialis, ſunt duæ lineæ quæ ſe mutuo tangunt. Formalis eſt alterius in alteram inclinatio. Unde ſequitur primò, quòd illæ duæ lineæ non ita ſe debent tangere, ut jaceant in directum, id eſt, ut unicam rectam conſtituant lineam; ſed altera debet in alteram inclinari.

Sequitur 2. quod anguli quantitas conſiſtit in majori vel minori linearum inclinatione, non in longitudine linearum.

Sequitur 3. non eſſe neceſſe, ut duæ lineæ poſt contactum productæ ſe mutuò ſecent, ut vult Pel-

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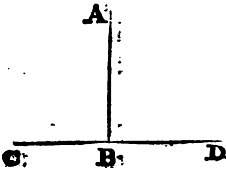
Pelletarius: id enim tantùm eſt verùm in angulis rectilineis: ſed ſufficere, ut ſe tangant & inclinentur.

Denique ſi angulus ille ſit in ſuperficie plana, dicetur planus. In omni vero figura, licet quemlibet angulum tribus litteris appellemus, ille tamen ſemper intelligitur, cui medius character appingitur.

9. *Cum autem continentes angulum lineæ rectæ fuerint, rectilineus appellatur angulus.*

Si utraque curua, curuilineus: ſi curua altera, altera recta; mixtus.

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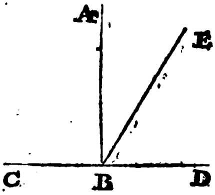


10. *Cum verò recta* A B. *ſuper rectam* C D. *ſtans, eos qui ſunt deinceps* A B C. A B D. *angulos, æquales inter ſe facit; rectus eſt uterque æqualium angulorum, & inſiſtens recta* A B. *perpendicularis vocatur ejus cui inſiſtit.* C D.

Tunc angulus uterque dicitur æqualis, quando recta A B. non magis in C. quam in D. inclinat.

Quod autem Græci dicunt κάθετος Latinè redditur perpendicularis; frequentius tamen utuntur Mathematici verbo Græco quam Latino, maximè in Optica: unde apud eos nihil uſitatius quàm ἄγειν κάθετον, imo Latine reddunt Cathetum.

11. *Ob-*



11. *Obtuſus angulus* EBC. *eſt, qui major recto* ABC.

Nempe quia recta EB. magis recedit à ſubjecta CD. quàm perpendicularis AB.

12. *Acutus vero* EBD. *qui minor recto* ABD.

13. *Terminus eſt quod alicujus eſt extremum.*

Talia ſunt, punctum, linea, ſuperficies: nempe punctum lineæ, linea ſuperficiei, & ſuperficies corporis.

14. *Figura eſt quæ ſub aliquo, vel ſub aliquibus terminis comprehenditur.*

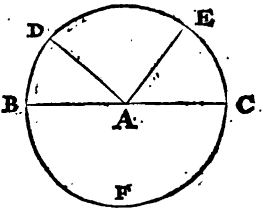
Dixit ſub aliquo, nempe quia circulum & ellipſim, unicus terminus, hoc eſt linea circularis, comprehendit: ad rectilineas verò figuras, plures ſemper termini requiruntur.

Porro notabis debere terminos, quantitatem, quæ figura dicitur, ambire & comprehendere, non vero tantum terminare. Unde ſequitur 1. Quod lineæ nulla proprie eſt figura, cum puncta lineam, non ambiant, ſed ſolum terminent. Sequitur 2. quod ſuperficiei infinitæ vel corporis infiniti; ſi quod dari poſſet, figura nulla ſit, 1. quia omnis figura debet ambire, & comprehendere figuratum. 2. quia terminis ambitur, terminus autem eſt extremum rei: Quomodo

vero

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vero id quod habet finem & extrema, erit infinitum?



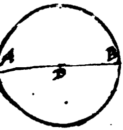
15. *Circulus eſt figura plana ſub una linea* A. B. C. *comprehenſa, quæ vocatur peripheria: ad quam ab uno puncto, eorum quæ intra figuram ſunt poſita, omnes cadentes rectæ* DA. AB. EA. AC. *æquales inter ſe ſunt.*

16. *Centrum vero circuli punctum illud appellatur.*

Theodoſius Sphæricorum lib. 1. deff. 1. & 2. idem habet, definitione vero 5. ſic polum deſcribit.

Polus

Polus circuli in Sphæra, eſt punctum in ſuperficie Sphæræ, à quo omnes rectæ ad circuli peripheriam tendentes, ſunt inter ſe æquales. Ex quibus colliges inter centrum, & polum hoc tantum eſſe diſcriminis, quod centrum concipiatur intra figuram poſitum: Polus vero in ſuperficie Sphæræ.



17. *Diameter autem circuli eſt recta quædam* AB. *per centrum* D. *ducta, & terminata ex utraque parte, à circuli peripheria* A. *&* B. *quæ & biſariam ſecat circulum.*

Hic tria obſervabis 1. omnes Diametros ejuſdem circuli eſſe æquales inter ſe, cum earum medietates ex def. 15. ſint æquales. 2. Quod ſequitur ex 1. eſt quod licet in circulo poſſint infinitæ

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duci

duci rectæ non tranſeuntes per centrum, ſolæ tamen rectæ per centrum ductæ, & in peripheria terminatæ dicuntur diametri, quia cum ſolæ ſint omnes æquales inter ſe, determinatæque longitudinis, aliæ vero inæquales ſemper & incertæ: diameter ſola poteſt metiri circulum. Menſura
4. enim cujuſque rei, ait Ptolomęus, in Analemmate, debet eſſe ſtata determinataque, non indefinita. Unde non eſt quod mirentur tyrones ſi in fœminino genere ponatur à Mathematicis. Idem enim eſt Diameter quod linea dimetiens, vel in duo æqualia dividens.

3. Eſt, Diametrum bifariam ſecare circulum, quod ita demonſtrat Thales apud Proclum. Concipe animo portionem ſemicirculi ſic coaptari portioni reliquæ ut diameter ſit utriuſque baſis. Si circumferentia una congruat penitus

nitus

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nitus circumferentiæ alteri, manifeſtum eſt illas duas portiones à diametro factas, eſſe inter ſe æquales, cùm neutra aliam excedat. Si verò circumferentia una non congruat cum altera, ſed vel extra eam cadat, vel intra, vel partim intra, partim extra: tunc rectæ ductæ à centro ad circumferentiam erunt æquales & non erunt.

18. *Semicirculus autem eſt figura quæ continetur ſub diametro* AB. *& ſub ea linea* ADB. *quæ aufertur de circuli peripheria.*

19. *Segmentum circuli eſt figura quæ continetur ſub recta & circuli peripheria.*

Per rectam hic intellige omnem non diametrum, niſi item velis ſemicirculum dicere ſegmentum.

20. *Recti*

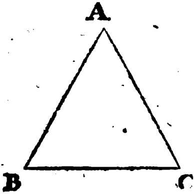
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20. *Rectilineæ figuræ ſunt quæ ſub rectis continentur.*

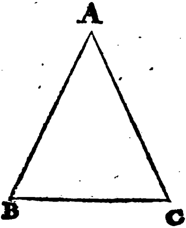
21. *Trilateræ quidem quæ ſub tribus.*

22. *Quadrilateræ verò quæ ſub quatuor.*

23. *Multilateræ autem quæ ſub pluribus quam quatuor rectis comprehenduntur.*

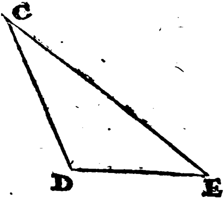


24. *Trilaterum porro figurarum, æquilaterum triangulum eſt quod tria latera habet æqualia.*



25. *Iſoſceles autem, quod duo tantum habet æqualia* A B. A C.

Σκέλος, τὸ, crus Græcis eſt, unde compoſitum ἰσοσκελὴς qui æqualibus eſt cruribus: τρίγωνον ἰσοσκελής; quod è tribus lineis duas æquales habet, quibus quaſi cruribus inſiſtit.

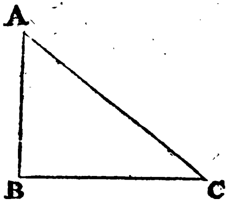


26. *Scalenum verò quod tria inæqualia habet latera.*

Triangulorum hæ ſunt ſpecies ex laterum ratione petitæ. Sequuntur aliæ ex angulorum differentiis emergentes.

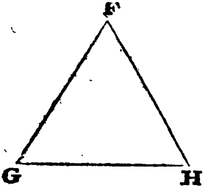
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27. *Ad*



27. *Ad hæc etiam trilaterarum figurarum, rectangulum quidem triangulum eſt quod habet rectum angulum* ABC.

28. *Amblygonium eſt quod habet obtuſum angulum, hoc eſt, majorem recto.*

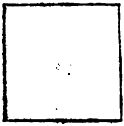


29. *Oxygonium vero quod tres acutos habet angulos, hoc eſt, minores recto.*

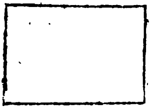
Not. In omni triangulo cujus duo quæcunque latera expreſsè no-

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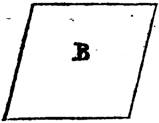
nominantur, ſolet reliquum latus à Mathematicis, baſis dici, ſive illud in ſitu locum infimum occupet, ſive ſupremum.



30. Quadrilaterum autem figurarum quadratum quidem eſt quod æquilaterum eſt & rectangulum.



31. Altera parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt.



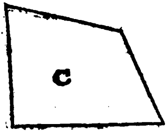
32. Rhombus autem, quæ æquilatera quidem, ſed rectangula non eſt.

33. Rhom-

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33. Rhomboides vero quæ adverſa, & latera, & angulos æqualia inter ſe habens, neque æquilatera eſt, neque rectangula.



34. Præter has autem reliquæ quadrilateræ, Trapezia appellantur.

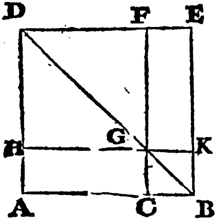
35. Parallelæ ſunt rectæ, quæ in eodem plano exiſtentes, & productæ in infinitum ex utraque parte, in neutram mutuo incidunt.

Ad hoc ut duæ rectæ dicantur parallelæ, non ſufficit ut productæ in infinitum non concurrant.

 Sic

Sic enim duæ rectæ in tranſverſum poſitæ re aliqua interpoſita, & non ſe tangentes, dicerentur parellelæ, quia nunquam concurrerent. Sed requiritur præterea, ut ſint in eodem plano.

36. *Parallelogrammum eſt figura quadrilatera, cujus bina oppoſita latera ſunt parallela ſeu æquidiſtantia.*



37. *Cum vero in parallelogrammo diameter* BD. *ducta fuerit, duæque rectæ* CF. HK. *lateribus parallelæ ſecantes diametrum in uno eodemque puncto* G. *ita ut parallelogrammum diſtri-*

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diſtributum ſit in quatuor parallelogramma; per quæ diameter non tranſit ſcil. AG. GE. *appellantur complementa eorum quæ circa diametrum conſiſtunt ut* HF. GK.

POSTULATA.

1. *Poſtuletur à quovis puncto* A. *ad quodvis punctum* B. *rectam lineam* AB. *ducere.*

2. *Et terminatam rectam* AB. *in continuum recta producere in* C.

3. *Et quovis centro, & intervallo circulum deſcribere.*

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Communes notiones ſeu Axiomata.

1. *Quæ eidem æqualia, & inter ſe ſunt æqualia.*

2. *Et ſi æqualibus æqualia adjecta ſint, tota ſunt æqualia.*

3. *Et ſi ab æqualibus æqualia ablata ſint, quæ relinquuntur ſunt æqualia.*

4. *Et ſi inæqualibus æqualia adjecta ſint, tota ſunt inæqualia.*

5. *Et ſi ab inæqualibus æqualia ablata ſint, reliqua ſunt inæqualia.*

6. *Et quæ ejuſdem duplicia, inter ſe ſunt æqualia.*

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7. *Et quæ ejuſdem dimidia, inter ſe ſunt æqualia.*

8. *Quæ congruunt ſibi mutuo, inter ſe æqualia ſunt.*

Id eſt, quæ collata, ita componuntur, ut pars parti reſpondeat, & terminus termino, æqualia ſunt. Lineæ autem rectæ & æquales congruunt, uti & anguli.

9. *Et totum parte majus eſt.*

10. *Et omnes anguli recti æquales inter ſe ſunt.*

11. *Si in duas rectas recta incidens interiores, & ad eaſdem partes angulos duobus rectis minores faciat; productæ duæ illæ rectæ in infinitum, coincident inter*

ſe

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ſe ad eas partes, in quibus ſunt anguli duobus rectis minores.

Scio principium hoc obſcurum quibuſdam, & à Gemino & Proclo rejectum à numero principiorum: verum non debet res aliqua à notionibus communibus rejici, quod unus aut alter ei aſſenſum neget: oporteret enim & nonum expungere. Jam enim ſunt aliqui Philoſophi adeo ſubtiles ut negent totum ſua parte majus. His & illis ſufficiat dicere Euclidem cæteroſque omnes, hæc omnia ex ſola terminorum notione, evidentia cenſuiſſe, & exiſtimaſſe ſenſu communi carere, qui ea negaret. Ne ſcrupulus remaneat, illud demonſtrat Clavius prop. 28. l. 1.

12. *Duæ rectæ ſpatium non comprehendunt.*

Id eſt ex omni parte concludunt.

13. *Omne totum eſt æquale omnibus partibus ſimul ſumptis.*

Plura talia axiomata excogitari poſſunt & ab aliis propoſita ſunt, ſed hæc ſufficere nullus dubito.

NOTA.

Quicquid proponitur vocatur propoſitio, eſtque vel problema vel Theorema.

Problema eſt propoſitio ubi aliquid proponitur efficiendum & concluſio ſemper talis eſt, quod erat faciendum.

Theo-

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Theorema eſt propoſitio cum proponitur proprietas vel veritas de aliqua re demonſtranda, & concluſionis formula. Quod erat demonſtrandum.

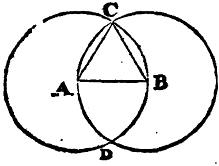
Quicquid autem tanquam conſectarium aut lucrum ex demonſtratione ſequitur Corollarium appellatur.

Lemma inſuper vocatur demonſtratio præmiſſæ alicujus, ut quæſiti demonſtratio evadat brevior ac clarior.

PRO-

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PROPOSITIO I.



Super data recta terminata A B. *triangulum æquilaterum* A B C. *conſtituere.* *Problema I.*

Praxis. Ex centris A & B. ſpatio A B. deſcribe [a] duos circulos, & ex puncto ſectionis C. duc [b] rectas C A. C B. Dico triangulum A B C. eſſe æquilaterum.

[a] 3. *Poſt.*
[b] 1. *Poſt.*

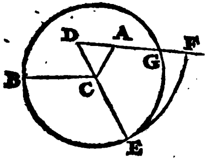
Probatur. Recta A C. æqualis eſt [c] rectæ A B. & B C. [c] eidem: ergo rectæ A C. B C. æquales eidem A B. æquales ſunt [d] inter ſe. Ergo triangulum A B C. eſt [e] æquilaterum. Quod erat faciendum.

[c] 15. *Def.*
[d] 1. *Ax.*
[e] 24. *Def.*

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PROPOSITIO II.

Prob. 2.



Ad datum punctum A. data recta B C. æqualem rectam A F. ponere.

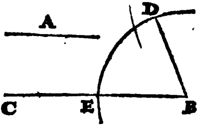
a 1. Poſt. Prax. Jungatur [a] A C. Super ipſa A C. fac [b] triangulum
b 1. 1. æquilaterum C D A. centro C.
c 3. Poſt. ſpatio B C. duc [c] circulum: latus D C. produc [d] in E. centro D.
d 2. Poſt. ſpatio D E. duc circulum: latus D A. produc in F. Recta A F. æqualis eſt rectæ C B.

Prob. Rectæ D A. D C. ſunt
e Ex conſt. [e] æquales. Rectæ D E. æqualis
f 15. Def. [f] recta D F. [g] Ergo recta A F. rectæ C E. Rurſum, recta [f] C E.
g 3. Ax. æqualis eſt rectæ C B. [h] Ergo
h 1. Ax. A F. ipſi C B. Quicunque autem alii ponantur caſus, eadem ſemper erit conſtructio & demonſtratio, ut bene notat Clavius ex Proclo.

PRO-

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PROPOSITIO III.



Datis duabus rectis inæqualibus A. *&* B C. *de majori* BC. *minori* A. *æqualem rectam* B E. *detrahere.* *Prob.* 3.

Prax. Ad datum punctum B. datæ rectæ A. æqualem rectam D B. [a] pono. Centro B. ſpatio B D. duco [b] circulum, abſciſſa B E. eſt æqualis ipſi A. a 2. 1. b 3. Poſt.

Prob. Recta B E. eſt [c] æqualis ipſi B D. quæ ponitur [d] æqualis ipſi A. Ergo abſciſſa B E. æqualis eſt [e] datæ A. Quod erat faciendum. c 15. Def. d Ex conſt. e 1. Ax.

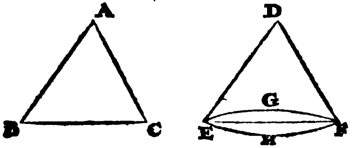
SCHOLIUM.

Circino hoc ut & præcedens problema fieri poteſt ſecundum Tacquet; ſed tunc ex ſententia Procli nullo poſtulato ſatisfacit.

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PROPOSITIO IV.

Theorema I.



Si duo triangula A. *&* D. *duo latera, duobus lateribus æqualia habeant utrumque utrique* hoc eſt A B. ipſi D E. & A C. ipſi D F. *habeantque angulos* A. *&* D. *lateribus illis contentos, æquales: Et Baſim* B C. *baſi* E F. *æqualem habebunt, & triangulum* A B C. *triangulo* D E F. *æquale erit, & reliqui anguli, reliquis angulis æquales erunt uterque utrique*, hoc eſt angulus B. angulo E. & angulus C. angulo F. æqualis erit, *ſub quibus æqualia latera* A B. ipſi D E. *&* A C. ipſi D F. *ſubtenduntur.*

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Prob. Latus A B. lateri D E. & latus A C. ipſi D F. & angulus A. angulo D. ponuntur æqualia: ergo ſi ſuperponantur, [a] congruent: ergo & baſis B C. baſi E F. congruet. Adeoque totum triangulum toti triangulo ſuper impoſito æquale erit. Q. E. D. a 8. Ax.

N O T A.

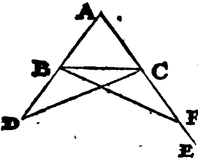
1. *Proprietas trianguli in hoc theoremate propoſita, cum ex terminorum explicatione videatur patere, poſſet aſſumi tanquam communis notio.*

2. *Quemadmodum duo latera cum angulo incluſo inferant æqualitatem baſium & angulorum; ſic & viciſſim dicendo, duo latera & baſes æquales inferre angulos æquales. Adeoque octava propoſitio tanquam conſectarium hujus haberi poterit.*

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PROPOSITIO V.

Theor. 2.



Iſoſcelis trianguli A B C. *qui ad baſim ſunt anguli* A B C. A C B. *inter ſe ſunt æquales, & productis æqualibus rectis* A B. A C. puta in D. & E. *qui ſub baſi ſunt anguli* C B D. B C F. *inter ſe æquales ſunt.*

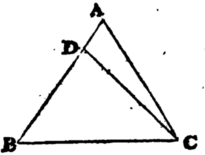
Præparatio. Ex lineis A B. A C. pro-
a 3. 1. ductis, accipio a æqualia B D. C F.
b 1. & b duco rectas C D. B F.
Poſt. Prob. Triangulorum B A F. C A D. unum latus B A. Uni C A. & alterum
c *Ex Hyp.* F A. alteri D A. c æquale eſt. Et angulus B A C. utrique eſt communis: ergo
d 4. 1. d angulus A B F. æqualis eſt angulo A C D. & angulus A F B. angulo A D C. & baſis B F. baſi C D. æqualis. Rurſus in triangulis B C D. C B F. latus C F. lãteri B D.
e 3. 1. e eſt æquale, & latus F B. probatum eſt æquale ipſi D C. & angulus D.
f 4. 1. angulo F. æqualis. Ergo f anguli C B D. B C F. infra baſim ſunt æquales & anguli B C D. C B F. æquales. Qui ſi tollantur ex æqualibus A B F. A C D. relinquent
g 3. *Ax.* angulos ad baſim g A B C. A C B. æquales. quod erat demonſtrandum. Thales fertur autor hujus propoſitionis.

Corollarium. Omne triangulum æquilaterum, eſt æquiangulum.

PRO-

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PROPOSITIO VI.



Si trianguli A B C. *duo anguli* A B C. A C B. *æquales inter ſe fuerint, & ſub æqualibus angulis ſubtenſa latera* A B. A C. *æqualia inter ſe erunt.* *Theor. 5.*

Si negas: pars unius B D. [a] fiat æqualis alteri C A. hoc poſito; triangula D B C. A C B. ſe habent juxta quartam; nam latus B C. commune, & latera B D. C A. æqualia, & anguli D B C. A C B. æquales. Ergo & totum triangulum æquale erit toti triangulo, hoc eſt totum parti: quod repugnat. [b]

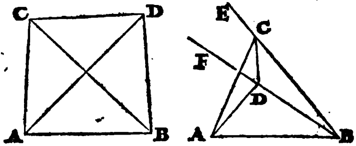
a 3. 1.

b 9. Ax.

Coroll. Omne triangulum æquiangulum, eſt æquilaterum.

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PROPOSITIO VII.



Theor. 4. *Super eadem recta* A B. *duabus eiſdem rectis* A C. B C. *æquales aliæ duæ rectæ* A D. B D. *utraque utrique*, hoc eſt A C. ipſi A D. & B C. ipſi B D. *non conſtituentur ad aliud & aliud punctum*, puta D. *ad eaſdem partes, eoſdem terminos* B. & A. *habentes, cum duabus initio ductis rectis.*

Prob. Quia ſi poſſint duci duæ aliæ, ducantur in D. Ergo triangulum C A D. [a] eſt Iſoſceles: ergo anguli A C D. A D C. æquales. Rurſus triangulum C B D. eſt Iſoſceles. Ergo anguli B D C. B C D. ſunt æquales, cùm tamen angulus C D A.

a 5. 1.

pars

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pars anguli totalis C D B. probatus ſit æqualis totali angulo A C D. Idemque ſequetur incommodum ubicumque ſtatuatur punctum verſus eaſdem partes. Nam ſi ponatur punctum intra triangulum in D. ut in ſecunda figura, ductis A D. B D F. B C E. & D C. ſic dico. Rectæ A D. A C. ponuntur æquales, ergo [b] anguli A D C. A C D. ſunt æquales: ſimiliter B D. B C. ponuntur æquales, ergo anguli infra baſim ECD. FDC. ſunt [b] æquales, ergo angulus F D C. major eſt angulo A D C. quemadmodum E C D. major eſt ipſo A C D. quod repugnat. b 5. 1.

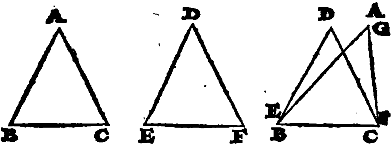
Denique non poteſt ſtatui punctum in parte alicujus lineæ ex datis, alioquin pars eſſet æqualis toti, contra 9. ax.

NB. *Hæc propoſitio tantum adhibetur ad demonſtrandam ſubſequentem octavam, quæ poſſet tamquam conſectarium quartæ aſſumi.*

PRO-

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PROPOSITIO VIII.



Theor. 5. *Si duo triangula* A. D. *duo latera*, A B. A C. *duobus lateribus* D E. D F. *æqualia habeant, alterũm alteri: habeant etiam baſim* B C. *baſi* E F. *æqualem: ... angulum* A. *angulo* D. *æqualem habebunt, ſub æqualibus rectis contentum.*

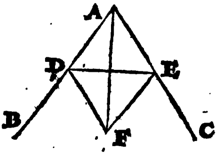
Prob. Quia ſi congruant latera, congruent & anguli: cùm [a] angulus non ſit aliud quàm inclinatio duarum linearum. Quod ſi quando ſuperponentur non congruant, ſed trianguli E F D. apex D. non cadat in A. ſed in G. ergo tunc duæ rectæ duabus rectis æquales, ſuper eadem recta B C. ducentur ad aliud punctum, contra præcedentem.

a 8. *Def.*

PRO-

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PROPOSITIO IX.



Datum angulum rectilineum B A C. *bifariam ſecare.* Prob. 4.

Prax. Ex lateribus dati anguli B A C. ſumo [a] rectam A D. & ipſi æqualem A E. Jungo D E. conſtituo [b] triangulum æquilaterum D E F. ducta recta A F. bifariam dividet angulum A.

a 3. 1.

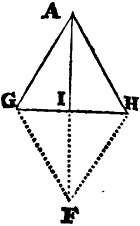
b 1. 1.

Prob. In triangulis D A F. E A F. rectæ A D. A E. ſunt æquales: A F. communis eſt, & baſis D F. baſi E F. æqualis: [c] ergo anguli F A D. F A E. ſunt æquales. Ergo angulus B A C. diviſus eſt bifariam. Quod faciendum erat.

c 8. 1.

PRO-

PROPOSITIO X.



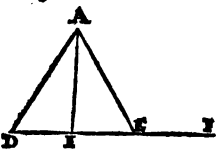
Prob. 5. *Datam rectam terminatam* G H.[a] *biſariam ſecare.*

a 1. 1. Prax. Supra rectam G H. [a] conſtituo triangulum æquilaterum G A H. cujus angulum
b 9. 1. A. divido [b] bifariam, ducta recta A F. dividet rectam G H. bifariam.

Prob. Triangula GIA. HIA. ſe habent juxta quartam ex conſtructione figuræ: ergo habent baſes G I. I H. æquales. Ergo recta G H. diviſa eſt bifariam. Q. E. F.

PRO-

PROPOSITIO XI.



Data recta DF. *à puncto* I. *in ea dato, ad rectos angulos, rectam lineam* I A. *excitare.* Prob. 6.

Prax. Ex linea D F. à puncto I. ſumo [a] partes hinc inde æquales I D. I E. ſuper D E. [b] conſtituo triangulum æquilatetum D A E. à puncto A. ad punctum I. recta ducta erit perpendicularis.

[a] 3. 1.
[b] 1. 1.

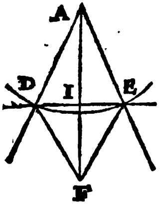
Prob. Latus D I. [c] eſt æquale lateri I E. & latus [d] D A. ipſi A E. & latus A I. commune. [e] Ergo anguli A I D. A I E. erunt æquales, [f] ergo recti: ergo [f] A I. perpendicularis. Q. E. F.

[c] *Ex conſt.*
[d] 23. *Def.*
[e] 8. 1.
[f] 10. *Def.*

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PROPOSITIO XII.

Prob. 7.



Super datam rectam infinitam DE. *à dato puncto* A. *quod in ea non eſt, perpendicularem rectam lineam* A I. *excitare.*

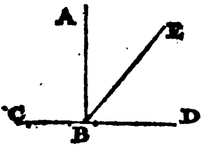
Prax. Centro A. duco circulum, qui ſecet rectam D E. à ſectionibus duco rectas D A. E A.
a 10. 1. [a] divido D E. bifariam in I. ducta recta A I. erit perpendicularis.

b 15. Def. c Ex conſt. d 8. 1. e 10. Def. Prob. Latera AD. AE. [b] ſunt æqualia, [c] latus D I. æquale lateri I E. & AI. commune: [d] ergo anguli A I D. A I E. ſunt æquales: [e] ergo recti: ergo A I. eſt [e] perpendicularis.

Hujus propoſitionis autor fertur Oenipides Chius annis ante Chriſtum circiter 550.

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PROPOSITIO XIII.



Cùm recta AB. vel EB. *ſupra rectam* CD. *conſiſtens, angulos facit: aut duos rectos* ABC. ABD. *aut duobus rectis æquales* EBC. EBD. *facit.* Theor. 6.

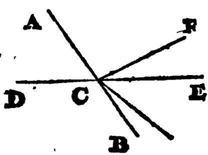
Prob. Recta EB. cum recta DC. aut facit utrinque æquales angulos & [a] conſequenter rectos; aut non facit: ſi non facit, [b] excitetur ex B. perpendicularis BA. Quoniam igitur angulo ABD. æquales [c] ſunt ABE. EBD. Si utriſque addas rectum ABC. [d] erunt duo recti ABC. ABD. æquales tribus angulis ABC. ABE. EBD. quibus etiam anguli EBC. EBD. ſunt æquales & conſequenter hi duo ſunt æquales duobus rectis. Q.E.D.

a 10. Def.
b 11. 1.
c 13. Ax.
d 2. Ax.

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PROPOSITIO XIV.

Theor. 7.



Si ad aliquam rectam A C. *& in ea punctum* C. *duæ rectæ* D C. C E. *non ad eaſdem partes ductæ, eos qui ſunt deinceps angulos* A C D. A C E. *duobus rectis æquales fecerint, in directum erunt inter ſe rectæ*, hoc eſt D C E. erit una linea recta.

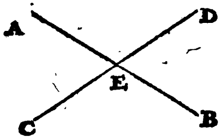
Prob. Si rectæ D C. C E. non jacent in directum, [a] jaceat C F. aut alia quæpiam. Ergo anguli A C D. A C F. valent [b] duos rectos. Ergo [c] pars A C F. eſt æqualis A C E. toti. Nam prius ex hypotheſi A C D. A C E. valebant duos rectos.

a Per 2. Poſt.
b 13. 1.
c Contra Ax. 9.

PRO-

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PROPOSITIO XV.



Si duæ rectæ Th. 8.
AB. CD. *ſe invicem ſecent, angulos ad verticem* AED. CEB. *æquales inter ſe facient.*

Prob. Nam angulo ſive AED. ſive CEB. addatur angulus medius DEB. a erit æqualis duobus rectis; b ergo anguli CEB. AED. ſunt æquales. Idemque fiet ſi angulo AEC. vel DEB. adjiciatur angulus AED.

a 13. 1.
b 3. Ax.

Thales Mileſius fertur auctor hujus propoſitionis.

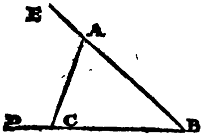
Coroll. 1. Duæ rectæ ſecantes ſe mutuo, efficiunt ad punctum ſectionis; quatuor angulos, quatuor rectis æquales.

Coroll. 2. Omnes anguli circa idem punctum conſtituti æquales ſunt quatuor rectis.

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PROPOSITIO XVII.



Trianguli Th. 10.
A B C. *duo anguli*, B C A. C A B. vel alii quilibet, *quocunque modo ſumpti, duobus rectis ſunt minores.*

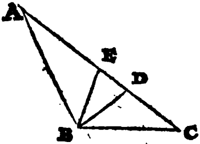
Prob. Producto B C. in D. externus angulus A C D. [a] major eſt angulo A. vel B. ſed a 16.1. anguli A C D. A C B. [b] valent b 13.1. tantùm duos rectos, ergo anguli B. & C. interni, ſive C A B. B C A. ſunt minores duobus rectis. Idem dicam de angulis A. & B. ſi producam latus, B A.

Coroll. 1. In omni triangulo, cujus unus angulus fuerit rectus vel obtuſus, reliqui ſunt acuti.

Coroll. 2. Omnes anguli trianguli æquilateri & trianguli Iſoſcelis, anguli ſupra baſim ſunt acuti.

PROPOSITIO XVIII.

Th. 11.



Trianguli A B C. *majus latus* A C. *majorem angulum* A B C. *ſubtendit.*

a 3. 1. b 5. 1. Si negas: Ex majori latere A C. [a] fac A D. æquale ipſi A B. duc rectam B D. [b] erunt anguli A B D. A D B. æquales. Eſt autem angulus A D B. hoc eſt A B D. externus & oppoſitus an-

c 16. 1. gulo C. [c] major. Multò ergo major eſt totalis angulus A B C. angulo C. Major item eſt angulo A. nam fac C E. æquale ipſi C B.

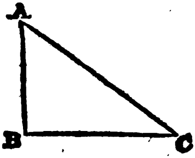
d 5. 1. [d] erunt anguli C E B. C B E.

e 16. 1. æquales, [e] & angulus C E B. hoc

f 9. Ax. eſt E B C. major angulo A. [f] ergo angulus A B C. major angulo A. Q. E. D.

PRO-

PROPOSITIO XIX.



Trianguli Tb. 12. A B C. *majus latus* A C. *ſub majori angulo* A B C. *ſubtenditur.*

Si negas latus A C. eſſe majus latere A B. ſint æqualia: [a] ergo anguli B. & C. ſunt æquales, contra hypotheſin. Si latus A B. dicas majus latere A C. [b] ergo angulus C. major erit angulo B. contra hypoth. Idem dicam de latere B C. Ex quibus ſic dico latus A C. nec minus eſt nec æquale lateribus AB. CB. ergo majus. Q. E. D.

a 5. 1.

b 18. 1.

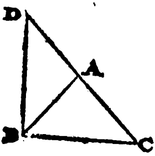
NOTA.

Hæc propoſitio eſt converſio præcedentis, quapropter hanc omittendo potuiſſet dici: ſi majus latus majorem angulum ſubtendit, utique & major angulus à majori latere ſubtenditur.

PRO-

PROPOSITIO XX.

Th. 13.



Trianguli ABC. *duo latera* puta AB. AC. *quomodocunque ſumpta, reliquo* BC. *ſunt majora.*

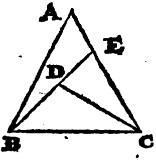
Prob. Produco CA. in D. ſic ut AD. ſit æquale ipſi AB. & proinde [a] CD. æqualis ipſis CA. AB. ducta recta DB. ſic dico: Rectæ AD. AB. ſunt æquales [b] ergo æquales anguli D. & DBA. [c] Major ergo utrolibet erit totus angulus DBC. ſed hunc angulum ſubtendit latus CD. hoc eſt CA. AB. [d] ergo rectæ CD. hoc eſt CA. AB. major eſt quàm latus BC. Q. E. D.

a 2. *Ax.*
b 5. 1.
c 9. *Ax.*
d 19. 1.

PRO-

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PROPOSITIO XXI.



Si ſuper trianguli ABC. *uno latere* BC. *ab extremitatibus duæ rectæ* BD. DC. *interius conſtitutæ fuerint, hæ conſtitutæ, reliquis trianguli duobus lateribus* AB. AC *minores quidem erunt, majorem verò angulum continebunt*, id eſt angulus D. major erit angulo A. Tb. 14.

Prob. 1. pars. Producto BD. in E. in triangulo BAE. duo latera BA. AE. a majora ſunt tertio BE. ergo a 20. 1. ſi addatur commune EC. erunt BA. AC. majora quam BE. EC. Eodem modo in triangulo CED. latera CE. ED. majora ſunt tertio CD. ergo ſi commune addatur DB. erunt CE. EB. majora quam BD. DC. ſed AB. AC. probata ſunt majora quam BE. EC. ergo multo majora quam BD. DC.

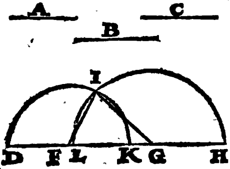
Prob. 2. pars. Angulus BDC. externus b major eſt interno & oppoſito DEC. b 16. 1. & hic major angulo A. interno & oppoſito, multo ergo major angulus BDC. angulo A. Q. E. D.

PRO-

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PROPOSITIO XXII.

Prob. 8.



Ex tribus rectis DF. FG. GH. *quæ ſunt æquales tribus datis rectis* A. B. C. *triangulum* FIG. *conſtituere; oportet autem duas quomodocunque ſumptas, reliqua eſſe majores :* a *quoniam omnis trianguli duo latera quomodocunque ſumpta reliquo ſunt majora.*

a 20. 1.

Prax. Datis rectis ABC. ſume ipſis ordine æquales DF.FG. GH. centro F. ſpatio FD. duc circulum DI. & centro G. ſpatio GH. duc alium HI. à puncto interſectionis I. ducantur rectæ FI. & GI. & factum eſt quod petitur.

Prob. in triangulo FIG. recta FI. æqualis eſt b ipſi DF. hoc eſt A. & GI. ipſi GH. hoc eſt C. & GF. ipſi B. Q. E. F.

b 15. Def.

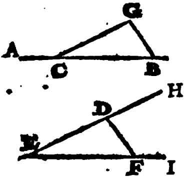
NOTA.

Hæc conditio in vigeſima propoſitione contenta omitti potuiſſet.

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PRO-

PROPOSITIO XXIII.



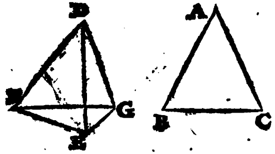
Ad datam rectam A B. *&* *punctum* C. *in ea datum, dato angulo rectilineo* D E F. *æqualem angulum rectilineum* G C B. *conſtituere.* Problema 9.

Sume in rectis E H. E I. duo puncta utcunque, puta D. & F. quæ recta D F. junges. Tum [a] fiat triangulum C G B. habens latera æqualia lateribus trianguli E D F. ſingula ſingulis: hoc facto triangula ſe habent juxta propoſitionem 8. ergo anguli E. & C. erunt æquales. Hujus propoſitionis autor fertur Oenipes Chius. a 22. 1.

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PROPOSITIO XXIV.

Th. 15.



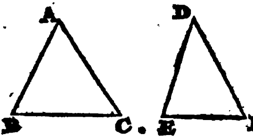
Si triãgulum ABC. *duo latera*, AB. AC. *duobus trianguli* DFE. *lateribus* DF. DE. *æqualia habuerit*, AB. *ipſi* DF. & AC. *ipſi* DE. *angulum vero* A. *majorem angulo* D. *baſim* BC. *baſi* FE. *majorem habebit*.

Ad rectam FD. & ad punctum
a 23. 1. in ea datum [a] ſiat angulus
FDG. æqualis angulo A. & latus DG. ipſi DE. hoc eſt ipſi
b 4. 1. AC. ſit æquale, [b] & conſequenter baſis BC. baſi FG. jungantur rectæ GE. GF. anguli DGE.
c 5. 1. DEG. [c] æquales erunt. Ergo totus angulus FEG. major quam DEG. major etiam erit quam DGE. & multo major quam
d 19. 1. FGE. [d] ergo recta GF. & huic æqualis BC. major eſt quam EF. Q. E. D.

PRO-

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PROPOSITIO XXV.



Si duo triangula A B C. D E F. *duo latera, duobus lateribus æqualia habuerint, alterum alteri* hoc eſt A B. ipſi E D. & A C. ipſi D F. *baſim verò* B C. *baſi* E F. *majorem habuerint: & angulum* A *angulo* D. *majorem habebunt ſub æqualibus rectis contentum.* Th. 16.

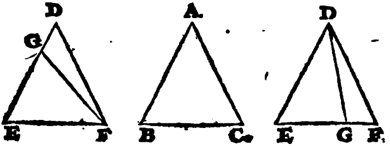
Prob. Quia ſi angulus A. non eſt major angulo D. erit vel æqualis, vel minor: ſi æqualis [a] ergo baſes B C. E F. erunt æquales, quod eſt contra hypotheſim. Si minor: cum latera A B. A C. ſint æqualia ipſis D E. D F. baſis E F. [b] major erit baſe B C. contra hypoth. ergo cum nec æqualis vel minor eſſe poteſt erit neceſſario major Q. E. D.

a 4. 1.

b 24. 1.

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PROPOSITIO XXVI.



Th. 17. *Si duo triangula, duos angulos, duobus angulis æquales habuerint, alterum alteri; & unum latus uni lateri æquale, ſive quod adjacet æqualibus angulis, ſive quod uni æqualium angulorum ſubtenditur, & reliqua latera, reliquis lateribus æqualia habebunt, alterum alteri, & reliquum angulum reliquo angulo.*

Prob. ſint in triangulis A B C. D E F. anguli B. & C. æquales angulis E. & F. ſintque primo latera B C. E F. (quæ adjacent angulis æqualibus) æqualia. Si latus E D. non eſt æquale ipſi B A. ſit eo majus, & ſumatur E G. æquale ipſi B A. tum ducta F G. Duo latera triangulorum G E F. A B C. æqualia ſunt, & anguli E. & B. æquales contenti inter latera æqualia. a Ergo anguli C. & G F E. ſunt æquales, quod eſſe non poteſt: nam angulus G F E. eſt pars ipſius D F E. qui æqualis ponebatur ipſi C. non ergo D E. major eſt quam B A. Sed neque minor, alias lateri B A. eadem quæ prius, applicaretur demonſtratio.

a 4. 1.

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ſtratio. Ergo æqualis. Ergo triangula D E F. A B C. ſe habent juxta 4. & latera lateribus, & anguli angulis correſpondentibus ſunt æquales.

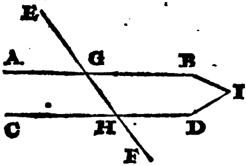
Sint deinde latera A B. D E. ſubtendentia æquales angulos C. & E F D. inter ſe æqualia, dico latera C B. C A. ipſis F E. F D. eſſe æqualia, & angulum A. angulo D. æqualem. Si enim latus E F ſit majus latere B C. ſume rectam E G. æqualem ipſi B C. duc rectam D G. quoniam igitur latera A B. B C. ſunt æqualia ipſis D E. E G. & anguli B. & E. ſunt æquales ex hypoth. erit b angulus C. angulo E G D. æqualis. b 4. 1.
Igitur & angulus E G D. angulo E F D. erit æqualis, hoc eſt externus interno & oppoſito c quod eſt abſurdum. Non c 16. 1.
eſt ergo latus E F. majus latere B C. ſed neque minus eſt, ut oſtendit eadem demonſtratio applicata lateri B C. ergo eſt ei æquale; ergo triangula A B C. D E F. ſe habent juxta 4. cum latus A B. ipſi D E. & B C. ipſi E F. & angulus B. angulo E. ſit æqualis & conſequenter baſis A C. baſi D F. Q. E. D.

Thales mileſius autor hujus fertur.

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PROPOSITIO XXVII.

Fb. 18.



Si in duas rectas AB. CD. *recta* EF. *incidẽs angulos alternos* AGH. DHG. *æquales inter ſe fecerit: parallelæ erunt inter ſe rectæ.*

a 35. Def.

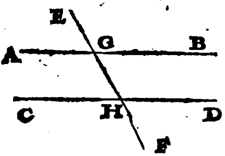
b 16. 1.

Prob. Si non ſunt parallelæ [a] coibunt tandem puta in I. & fiet triangulum GIH, cujus angulus externus AGH. erit [b] major interno & oppoſito GHD. cui tamen ex hypotheſi erat æqualis. Similiter demonſtrabitur, ſi dicantur concurrere in K. Ergo non concurrunt. Ergo ſunt parallelæ Q. E. D.

PRO-

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PROPOSITIO XXVIII.



Si in duas Th. 19.
rectas AB. CD. *recta* EF. *incidens, externum angulum* AGE. *interno & oppoſito & ad eaſdem partes* GHC. *æqualem fecerit: aut internos & eaſdem partes* AGH. GHC. *duobus rectis æquales fecerit: parallelæ erunt inter ſe rectæ.*

Prob. 1. pars. Angulo AGE. a æqualis eſt angulus BGH. angulus CHG. æqualis ponitur angulo AGE. b ergo alterni BGH. GHC. ſunt æquales, c ergo rectæ AB. CD. ſunt parallelæ.

a 15. 1.
b 1. Ax.
c 27. 1.

Prob. 2. Angulus EGA. cum angulo AGF. d valet duos rectos, anguli AGH. GHC. ponuntur æquales duobus rectis: e ergo ſubducto communi angulo AGH. remanebunt anguli EGA. GHC. æquales. Ergo rectæ AB. CD. ſunt parallelæ per priorem partem hujus.

d 13. 1.
e 3. Ax.

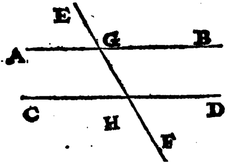
Ex ſecunda parte hujus propoſitionis, conſtat ſufficienter de veritate undecimi Axiomatis: nimirum à contrario.

NOTA.

Hæ tres proprietates 27. *ac* 28. *propoſitione propoſitæ unicâ contineri potuiſſent uti ſequens* 29. *quæquæ eatenus per modum converſionis demonſtrata videtur.*

PROPOSITIO XXIX.

Th. 20.



In parallelas rectas A B. C D. *recta* E F. *incidens: & alternos angulos* B G H. G H C. *æquales inter ſe facit: & externum* E G B. *interno & oppoſito & ad eaſdem partes* E H D. *æqualem: & internos ad eaſdem partes* A G H. C H G. *duobus rectis æquales.*

Prob. 1. pars. Anguli D H G.
a 13. 1. G H C. [a] valent duos rectos:
anguli item D H G. B G H.
b 28. 1. [b] valent duos rectos [c] ergo ſub-
c 3. Ax. ducto communi angulo D H G.
anguli B G H. G H C. alterni remanebunt æquales.

Prob. 2. Anguli E G B.
d 13. 1. B G H. valent [d] duos rectos: anguli B G H. G H D. valent [e] duos

duos rectos, ergo ſubducto com- e 28. 1.
muni B G H. remanebunt anguli
E G B. E H D. æquales.

Prob. 3. Rectæ A B. C D. ponuntur parallelæ f ergo neque verſus A. neque verſus B. concurrunt, ergo tam verſus A. quam verſus B. anguli interni ad eaſdem partes ſunt æquales duobus rectis, g ſi enim ex aliqua parte eſſent minores, ex ea concurrerent.

f 35. Def.

g 11. Ax.

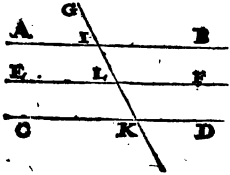
Coroll. Omne parallelogrammum, habens unum angulum rectum, eſt parallelogrammum rectangulum.

PRO-

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PROPOSITIO XXX.

Th. 21.



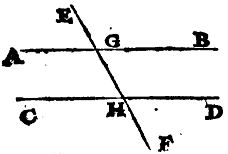
Quæ eidem recta E F. *parallela* A B. C D. *& inter ſe ſunt parallela.*

Prob. In has tres rectas in eodem plano poſitas ſi cadat recta G K. angulus A I L. æqualis erit angulo I L F. [a] quia ſunt alterni; & angulus externus I L F. angulo L K D. interno & oppoſito: [b] ergo anguli A I L. L K D. ſunt æquales: [c] ergo rectæ A B. C D. ſunt parallelæ Q. E. D.

a 29. 1.
b 1. Ax.
c 27. 1.

PRO-

PROPOSITIO XXXI.



A dato puncto G. data recta C D. parallelam rectam lineam A B. ducere. Problema 10.

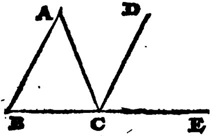
Ex G. in datam C D. duc rectam G H. utcunque, & angulo. G H D. [a] conſtituatur æqualis ad G. nempe angulus H G A. [b] erit recta A B. ipſi C D. parallela, quia anguli alterni A G H. D H G. ſunt æquales Q. E. F.

a 23. 1.

b 27. 1.

PRO-

PROPOSITIO XXXII.

Fi. 22. 

Trianguli A B C. *uno latere* B C. *producto* in E. *externus angulus* A C E. *duobus internis & oppoſitis* A B C. B A C. *aqualis eſt: & trianguli, tres interni anguli* A. B. C. *duobus rectis aquales ſunt.*

a 31. 1. Prob. 1. pars. [a] Ducatur ex C. recta C D. parallela rectæ A B. tunc quia recta A C. cadit in parallelas A B. C D. angulus
b 29. 1. A. æqualis eſt [b] alterno A C D. Et quia B C. cadit in eaſdem, parallelas angulus E C D. externus
c 29. 1. [c] æqualis eſt interno B. Totalis ergo A C E. æqualis eſt duobus internis & oppoſitis A. & B. Q. E. D.

Prob. 2. Angulus A C B.
d 13. 1. cum externo A C E. [d] valet duos rectos,

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rectos, ſed angulus A C E. [e] æqualis eſt angulis A. & B. ergo angulus C. cum angulis A. & B. valent duos rectos, ergo tres anguli, &c. Hujus propoſitionis autor fertur Pythagoras Samius circa annum ante Chriſt. 650. e 32.1.

Coroll. 1. Omnes tres anguli vnius trianguli, ſunt æquales tribus cujuſcunque alterius trianguli ſimul ſumptis; & quando duo ſunt æquales duobus, erit & reliquus reliquo æqualis.

Coroll. 2. In triangulo Iſoſcele rectangulo, anguli ad baſim ſunt ſemirecti.

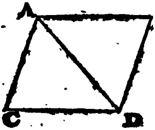
Coroll. 3. Angulus trianguli æquilateri eſt una tertia duorum rectorum, vel duæ tertiæ unius recti.

Sch. Omnis figura rectilinea diſtribuitur in tot triangula, quot ipſa continet latera, demptis duobus, & anguli triangulorum, conſtituunt angulos figuræ.

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PROPOSITIO XXXIII.

Th. 23.



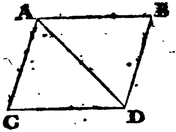
Recta A C. B D. *quæ æquales & parallelas* A B. C D. *ad eaſdem partes conjungunt: & ipſæ æquales & parallelæ ſunt.*

Prob. Duc rectam D A. quæ datas A B. C D. jungat [a] tunc
a 29. 1. anguli alterni D A B. A D C. erunt æquales: latus A B. ponitur æquale lateri C D. latus A D. eſt commune ergo baſes A C.
b 4. 1. D B. ſunt æquales. [b] Ergo anguli C A D. A D B. ſunt æqua-
c 27. 1. les; [c] ergo rectæ A C. D B. ſunt parallelæ.

PRO-

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PROPOSITIO XXXIV.



Parallelogrammorum ſpatiorum quæ ex adverſo & latera AB. CD. AC. BD. *&* *anguli* A. *&* D. B. *&* C. *æqualia ſunt inter ſe, & diameter* AD. *illa bifariam ſecat.* Th. 24.

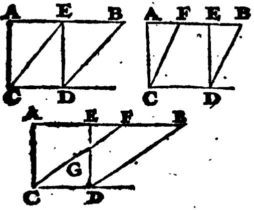
Prob. Rectæ AB. CD. ponuntur parallelæ, [a] ergo angulus BAD. angulo CDA. & angulus CAD. angulo ADB. ſunt æquales, cum ſint alterni. Ergo triangula ABD. ACD. habent duos angulos æquales alterum alteri, & ipſis commune latus AD. adjacet; [b] ergo & reliqui anguli B. & C. ſunt æquales, & reliqua latera, AB. ipſi CD. & BD. ipſi AC. erunt æqualia, cum æqualibus angulis, nempe alternis opponantur. [c] Ergo triangula ABD. ACD. æqualia inter ſe ſunt. Q. E. D.

a 29. 1.
b 26. 1.
c 4. 1.

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PROPOSITIO XXXV.

Th. 25.



Parallelogramma AD.FD. *ſuper eadem baſi* CD. *& in iiſdem parallelis* AB. CD. *conſtituta, inter ſe ſunt æqualia.*

Id tribus modis poteſt contingere, ſi, ut vides, in 1. figura, ſic dico. Rectæ AE. EB. ſunt
a 1. Ax. [a] æquales, quia ſunt [b] æquales
b 34. 1. rectæ CD. Rectæ AC. ED. ſunt æquales: angulus CAE.
c 29. 1. [c] æqualis eſt angulo DEB. ergo
e 4. 1. triangulum CAE. [e] æquale eſt
f 2. Ax. triangulo DEB. [f] addito ergo communi FCD. fient parallelogramma AECD. CEBD. æqualia.

Si ut in 2. Rectæ AE. FB.
g 3. Ax. ſunt æquales ut prius: [g] dempta igitur communi FE. erunt æqua-

les

les A F. E B. Rectæ A C. E D. ſunt [h] æquales: anguli A. & E. h 34. 1.
ſunt [i] æquales, [l] ergo triangula i 29. 1.
FAC. BED. ſunt æqualia, addito l 4. 1.
ergo communi trapeſio E F C D. parallelogramma A E C D. F B C D. erunt [m] æqualia. m 2. Ax.

Si ut in 3a. idem repeto. Rectę AE. FB. ſunt [n] æquales ipſi C D. n 34. 1.
[o] ergo & inter ſe: ergo recta A F. o 1. Ax.
[p] æqualis eſt rectæ E B. Rectæ p 2. Ax.
A C. E D. ſunt [q] æquales, anguli item E. & A. ſunt [r] æquales: ergo triangula A C F. E D B. ſunt q 34. 1. r 29. 1.
[ſ] æqualia: Ergo ſi ab utroque tollas triangulum E G F. relinques ſ 4. 1.
æqualia trapezia A C G E. & F G D B. quibus ſi addas commune triangulum C G D. facies parallelogramma A D. D F. æqualia. Q. E. D.

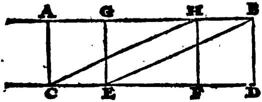
SCHOLIUM.

Hinc omnium parallelogrammorum dimenſio, cum æqualia ſint parallelogrammo rectangulo, cujus area provenit ex multiplicatione laterum, patebit.

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PROPOSITIO XXXVI.

Th. 26.



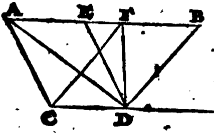
Parallelogramma A E. H D. *ſuper æqualibus baſibus* C E. F D. *& in iiſdem parallelis* A B. C D. *conſtituta, inter ſe ſunt æqualia.*

Prob. Connectantur parallelogramma rectis C H. E B.
a 34.1. [a] quæ erunt æquales & parallelæ. Hoc poſito, parallelogrammum
b 35.1. A E. æquale eſt ipſi [b] C B. & parallelogrammum CB. ipſi [b] HD.
c 1. Ax. [c] ergo parallelogramma A E. H D. ſunt æqualia. Q. E. D.

PRO-

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PROPOSITIO XXXVII.

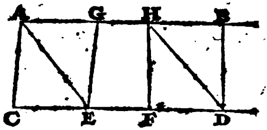


Triangula ACD. FCD. *ſuper eadem baſi* CD. *&* *in iiſdem parallelis* AB. CD. *conſtituta, ſunt inter ſe æqualia.* Tb. 27.

Prob. [a] Per D. ducas D E. parallelam rectæ C A. & D B. ipſi G F. parallelogramma A D. C B. [b] erunt æqualia: [c] ſed eorum dimidia ſunt triangula A C D. F C D. [d] ergo ipſa triangula A C D. F C D. ſunt æqualia. Q. E. D.

a 31. 1.
b 35. 1.
c 34. 1.
d 7. Ax.

PROPOSITIO XXXVIII.

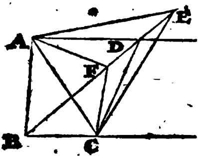


Th. 28. *Triangula* A C E. B F D. *ſuper æqualibus baſibus* C E. F D. *& in iiſdem parallelis* A B. C D. *æqualia ſunt inter ſe.*

a 31. 1. Prob. a Ducatur E G. parallela ipſi A C. & F H. ipſi B D.
b 36. 1. b erunt parallelogramma C G.
c 34. 1. H D. æqualia. c Horum dimidia ſunt triangula A G E. B F D.
d 7. *Ax.* d Ergo ſunt inter ſe æqualia. Q. E. D.

PRO-

PROPOSITIO XXXIX.



Æqualia triangula A B C. D B C. *ſuper eadem baſi* BC. *& ad eaſdem partes conſtituta, in iiſdem ſunt parallelis.* Hoc eſt A D. eſt parallela B C. Th. 29.

Prob. Si negas ſit. [a] A E. ipſi B C. parallela cui recta B D. producta occurrat in E. Ducta ergo recta C E. [b] triangula ABC. EBC. erunt æqualia, pars toti, quod fieri nequit: nam triangulum D B C. æquale triangulo A B C. æquale quoque foret triangulo E B C. per 1. ax. Quod ſi dicas A F. & B C. eſſe parallelas, eadem repetetur demonſtratio, & ſequetur totum & partem eſſe æqualia.

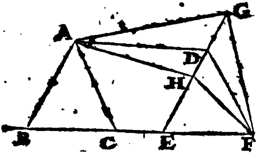
a 31. 1.

b 37. 1.

PRO-

PROPOSITIO XL.

Th. 30.



ᵃÆqualia triangula A B C. D E F. *ſuper æqualibus baſibus* B C. E F. *& ad eaſdem partes conſtituta, in iiſdem ſunt parallelis* A D. B F.

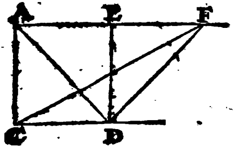
Prob. Si negas A D. ipſi B F. eſſe parallelam; ſit A G. cui occurrat E D. producta in G. Tunc ducta G F. erunt ᵃ triangula G E F. A B C. æqualia: ponebantur autem æqualia triangula ABC. DEF. ergo totum G E F. & pars D E F. eidem triangulo A B C. æquale; ᵇ erunt æqualia. Quod fieri nequit.

a 38. 1.

b 1. *Ax.*

PRO-

PROPOSITIO XLI.



Si parallelogrammum AECD. *commune cum triangulo* FCD. *baſim* CD. *habuerit, & in iiſdem parallelis* AF. CD. *fuerit: parallelogrammum erit duplum trianguli.* Th. 31.

Prob. Ducatur diameter AD. Triangula FCD. ACD.
a ſunt æqualia; Parallelogram- a 37. 1.
mum CE. b eſt duplum trianguli b 34. 1.
ACD. c ergo & trianguli FCD. c 6.
Q. E. D. *Ax.*

SCHOLIUM.

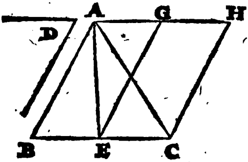
Cum jam per 35. omnium parallelogrammorum area obtinetur, etiam triangulorum, quæ eorundem dimidia ſunt, non latebit.

PRO-

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PROPOSITIO XLII.

Proble-ma II.



Dato triangulo A B C. *æquale parallelogrammum* G C. *conſtituere in dato rectilineo ang.* D.

Dati trianguli A B C. Baſim B C. divide [a] bifariam in E. ductaque E A. [b] agatur per A. recta A H. parallela ipſi B C. Ad punctum E. [c] facto angulo G E C. ipſi D. æquali; educatur ex C. recta C H. ipſi E G. [d] parallela dico factum.

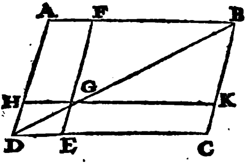
a 10. 1.
b 31. 1.
c 23. 1.
d 31. 1.

Prob. Triangula A B E. A E C. ſunt [e] æqualia: triangulum A E C. eſt dimidium trianguli, A B C. & [f] dimidium parallelogrammi B C. ſuper eadem baſi E C. conſtituti: ergo triangulum A B C. eſt [g] æquale parallelogrammo G C. habens ex conſtructione angulum G E C. æqualem dato angulo D. Q. E. F.

e 38. 1.
f 41. 1.
g 6. *Ax.*

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PROPOSITIO XLIII.



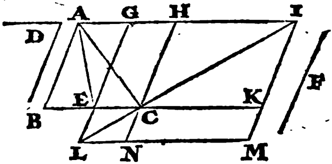
Omnis parallelogrammi, complemẽta eorum quæ circa diametrum ſunt parallelogrammorum, inter ſe ſunt æqualia. Th. 32.

In hac figura, parallelogramma circa diametrum ſunt, FK. HE. complementa verò eorum, parallelogramma AG. GC. hæc complementa dico eſſe æqualia.

Prob. Triangula BAD. BCD. ſunt [a] æqualia. Itemque triangula BKG. BFG. & GED. GHD. Ergo ſi ab æqualibus triangulis BAD. BCD. tollas æqualia, nempe BKG. ipſi BFG. & GHD. ipſi GED. complementa GA. GC. quæ remanent, erunt æqualia. Q. E. D. a 34. 1.

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PROPOSITIO XLIV.



Proble-ma 12. *Ad datam rectam* F. *dato triangulo* A B C. *æquale parallelogrammum* C M. *applicare in dato angulo rectilineo* D.

Conſtitue triangulo A B C.
a 42. 1. [a] æquale parallelogrammum
C G. habens angulum G E C.
æqualem angulo dato D. tum
produc B C. in K. ſic ut C K.
b 3. 1. ſit [b] æqualis datæ F. per K. agatur
c 31. 1. [c] K I. parallela ipſi C H. occurrens G H. productæ in I. Deinde ex I. ducatur per C. diameter I C. occurrens rectæ G E. productæ in L. & per L. ducatur L M. parallela ipſi E K. ſecans I K.

pro-

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productam in M. producaturque HC. in N. dico parallelogrammum CM. eſſe quod petitur.

Prob. Complementa GC. CM. ſunt [d] æqualia, parallelogrammum GC. eſt [e] æquale triangulo ABC. ergo & complementum CM. habet autem lineam CK. æqualem datæ F. & angulum CNM. æqualem angulo HCK. qui [f] æqualis eſt angulo GEC. qui ponitur æqualis dato angulo D. ergo parallelogrammum CM. æquale eſt triangulo ABC. & habet lineam CK. æqualem datæ F. & angulum GNM. æqualem dato D. Q. E. F.

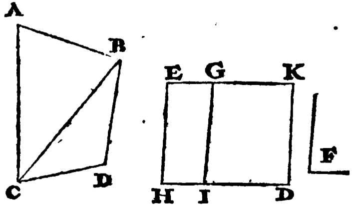
d 44. 1.

e Ex conſt.

f 28. 1. Prop.

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PROPOSITIO XLV.



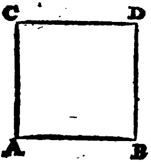
Proble-ma 13. *Dato rectilineo* A D. *æquale parallelogrammum* E D. *conſtituere, in dato rectilineo angulo* F.

a 44. 1. Divide rectilineum in triangula, fac parallelogrammum a E I. æquale triangulo B C D. in angulo H. æquali ipſi F. & ſupra latus G I. parallelogrammum G D. æquale triangulo A B C. habens in I. angulum G I D. æqualem ipſi H. & factum eſt quod petitur.

b Ex conſt. Prob. Parallelogrammum E I. æquale eſt b triangulo B C D. in angulo H. æquali dato F. rurſus parallelogrammum G D. æquatur triangulo A B C., etiam in angulo dato, ergo parallelogrammum E D. quod æquale eſt partibus ſimul ſumptis, æquatur rectilineo A B C D. in dato angulo Q. E. F.

PRO-

PROPOSITIO XLVI.



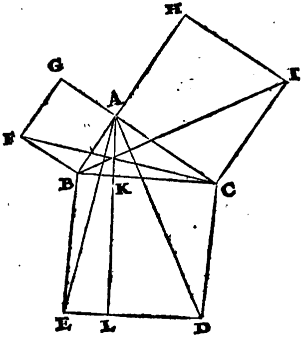
Datâ rectâ AB. *quadraum* ABDC. *deſcribere.* *Proble-ma 14.*

Ex A. & B. [a] erige perpendiculares C A. D B. æquales ipſi AB. jungaturque recta C D. & factum eſt quod petitur. a 11. 1.

Prob. [b] Anguli A. & B. ſunt recti: ergo rectæ A C. B D. ſunt [c] parallelæ. Utraque [d] eſt æqualis ipſi AB. ergo & inter ſe: [e] ergo & AB. & CD. parallelæ, ſunt æquales: ergo AC. CD. DB. ſunt æquales, & figura eſt parallelogramma: cumque anguli A. & B. ſint recti, [f] erunt etiam oppoſiti C. & D. recti. Ergo ABDC. eſt quadratum. Q. E. F. b *Ex conſt.* c 28. 1. d *Ex conſt.* e 33. *Prop.* f 34. 1.

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PROPOSITIO XLVII.



Th. 33. *In rectangulo triangulo* B A C. *quadratum* BD. *quod à latere* BC. *rectum angulum* B A C. *ſubtendente deſcribitur; æquale eſt quadratis* B G. G H. *quæ à lateribus* B A. AC. *rectum angulum* BAC. *continentibus, deſcribuntur.*

a 31. 1. Prob. Ex puncto A. duc [a] rectam A L. parallelam ipſi B E. & junge rectas, A D. B I. Triangula A C D. I C B. ſe habent juxta 4.nam latera CD. CA. ſunt

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ſunt æqualia ipſis C B. C I. & anguli contenti ICB. ACD. æquales: cum anguli I C A. B C D. ſint [b] recti & angulus A C B. communis: ergo triangula ACD. BCI. ſunt æqualia. Sed triangulum ACD. eſt [c] dimidium parallelogrammi L C. cum ſint ſupra eandem baſim C D. & inter eaſdem parallelas AL. CD. & triangulum I C B. dimidium eſt quadrati CH. ob eandem cauſam. [d] Ergo quadratum CH. eſt æquale parallelogrammo L C. cum eorum dimidia ſint æqualia.

b 30. Def.

c 41. 1.

d 6. Ax.

Jam ducantur rectæ A E. F C. Triangula F B C. A B E. ſunt æqualia, cum ſe habeant juxta 4. & triangulum A B E. eſt dimidium parallelogrammi B L. ſicut triangulum F B C. dimidium quadrati B G. ergo quadratum B G. eſt æquale parallelogrammo B L. Totum ergo quadratum BD. æquale eſt quadratis BG. CH. Q. E. D.

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SCHOLIUM.

Nobiliſſimum hoc Pythagoræ inventum præter infinitas utilitates, quas per univerſam Matheſin ſpergere nemo inficias ibit, Methodum nobis tradit.

1. *Ex datis duobus quibuſcunque lateribus in triangulo rectangulo reliquum latus invenire. Nimirum ſi* AB. 6. *partium* AC. 8. *erit* BC. 10. *nam ſi quadratum* AB. 36. *addatur ad quadratum* AC. 64. *ſumma erit* 100. *ex quo extracta radix erit* 10. *ipſum latus quæſitum* BC.

Vel ſi BC. *ſit* 10. AB. 6. *erit* AC. 8. *quoniam ſi à quadrato* BC 100. *ſubtrahatur quadratum* AB. 36. *relinquitur* 64. *cujus radix eſt latus quæſitum* AC.

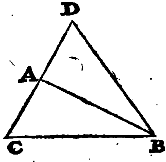
2. *Additionem & ſubtractionem quadratorum, quæ differentiam inter datarum linearum quadrata oſtendit.*

3. *Cum ex tribus rectis lineis* 3. 4. 5. *partium vel ex æquo per alios numeros multiplicatis, non niſi triangulum rectangulum conſtitui poteſt (quod occaſionem Pythagoræ de hoc invento dediſſe plurimi contendunt) in ipſis campis ſemper poterimus funiculo conficiens jam dictum triangulum pythagoricum, angulum rectum determinare.*

PRO-

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PROPOSITIO XLVIII.



Si quadratum quod Th. 34.
à C B. *uno laterum trianguli* CAB. *deſcribitur, æquale ſit iis quæ à reliquis duobus trianguli lateribus* AB. AC. *deſcribuntur quadratis: angulus* C A B. *contentus ſub reliquis duobus trianguli lateribus* A B. A C. *rectus eſt.*

Prob. [a] Ducatur ex A. ipſi AB. perpendicularis AD. ipſi AC. æqualis, jungaturq; recta DB. hoc poſito ſic dico. Angulus D A B. [b] rectus eſt, [c] ergo quadratum rectæ D B. æquale eſt quadratis rectarum AB. AD. vel AC. Sed quadratum ipſius C B. ex hypoth. æquale eſt quadratis earundem CA. AB. [d] ergo rectæ CB. B D. ſunt æquales. Ergo triangula C A B. ADB. habent tria latera æqualia, [e] & angulos qui æqualibus lateribus reſpondent æquales. Ergo ſi angulus D A B. rectus eſt, erit etiam rectus C A B. cum latera D B. B C. ſint æqualia. Q. E. D.

a 11. 1.
b Ex conſt.
c 47. 1.
d 1. Ax.
e 8. 1.

SCHO-

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SCHOLIUM.

Quamquam omnes propoſitiones in libris Euclidis ſuam per Univerſam Matheſin obtineant Uſum, nihilominus ob frequentiorem allegationem, quaſdam eſſe ſeligendas nullus dubito, quarum catalogum, ut hic, poſt omnes ſequentes, apponam libros.

Libri primi Inſigniores propoſitiones. 4. 5. 6. 13. 15. 26. 29. 31. 32. 36. 37. 38. 41. 47. *quibus à nonnullis annumerantur* 18. 19. 20.

Problemata porro paſſim per totum librum primum diſperſa, ad exercitium regulæ ac circini minimè negligenda ſunt; cum in ſubſequentibus conſtructionum facilitatem pareant.

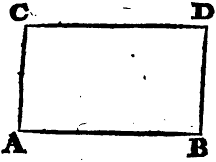
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EVCLIDIS ELEMENTUM II.

DEFINITIONES

I.



Parallelogrā-mum rectangulū ABCD. *contineri dicitur ſub duabus rectis* AB. BD. *quæ rectum angulum* ABD. *comprehendunt.*

Quemadmodum in circulo cognita diametro, tota ejus area cognoſcitur, ſic expreſſis duabus lineis quæ angulum rectum continent in parallelogrammo rectangulo, ſtatim tota ejus quantitas intelligitur, nimirum latitudo & longitudo.

Obſerva 1. Illud parallelogrammum dici rectangulum quod unum

unum habet angulum rectum. Si enim unus eſt rectus [a] [b] erunt & reliqui recti.

a 29. 1.
b 34. 1.

Obſerva 2. In ſequentibus nomine rectanguli, Euclidem ſemper intelligere parallelogrammum rectangulum, licet vis nominis id non exigat.

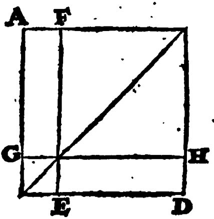
3. Geometras omne parallelogrammum exprimere duas tantum nominando literas, quæ per diametrum opponuntur. Ut appoſitum parallelogrammum appellant. A D.

4. Cognitis lateribus rectanguli, inveniri ejus aream ex multiplicatione numeri unius lateris in numerum alterius lateris circa eundem angulum. Similiterque cognita area rectanguli & uno laterum, inveniri alterum latus ſi dividatur numerus areæ per numerum lateris dati, quotiens enim erit latus quæſitum.

II.

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II.



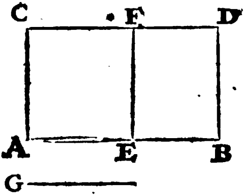
Omnis parallelogrāmi ſpatii unumquodlibet eorum quæ circa diametrū illius ſunt, parallelogrāmorum, cum duobus complementis, gnomon vocetur.

In parallelogrammo A D. parallelogrammum G E. cum duobus complementis G E. E H. vocetur γνομῶν, quod Latinè normam ſonat, ejus enim ſpeciem nobis exhibet.

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PROPOSITIO I.

Th. 1.



Si fuerint duæ rectæ G. *&* A B. *ſeceturque altera ipſarum* A B. *in quotcunq; ſegmẽta* A E. E B. *rectangulum* CB. *comprehenſum ſub duabus rectis* AC. *inſectâ* hoc eſt G. & A B. *ſectâ, æquale eſt rectangulis* C E. F B. *quæ ſub inſectâ* C A. *&* *quolibet ſegmentorum* A E. E B. *comprehenduntur.*

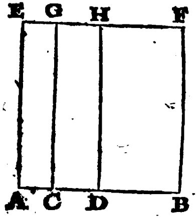
a 11. & 3. 1. Prob. Ex punctis A. & B. erige a perpendiculares AC. BD. æquales datæ G. & ducatur recta C D. ſicque fiat ex lineis C A, hoc eſt G. & A B. rectangulum C B. Rectam A B. utcunque di-
d 31. 1. vide in E. & fiat d E F. parallela & æqua-
& 3. 1. lis ipſi A C. erunt C E. F B. rectangula.
e 29. 1. Nam angulus F E B. rectus eſt e quia
f 28. 1. æqualis ipſi A. & conſequenter f reliqui
g 34. 1. anguli recti, & latera g lateribus oppoſitis æqualia. Hæc autem duo rectangula C F. B F. ſimul ſumpta ſunt æqualia totali B C. hoc eſt partes toti. Q. E. D.

Idem patet in numeris, puta 6. & 2. divide 6. in 2. & 4. dico 12. numerum productum ex 6. in 2. ęqualem eſſe duobus numeris 4. & 8. qui fiunt ex multiplicatione duorum in duo, & in quatuor.

PRO-

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PROPOSITIO II.

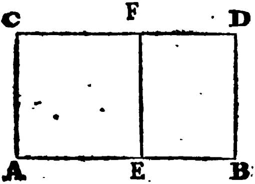


Si recta linea Th. 2.
A B. *ſecta ſit utcunque* puta in C. & D. *Rectangula* E C. G D. H B. *comprehenſa ſub tota* A E. hoc eſt A B. *&* *quolibet ſegmentorum* A C. C D. B D. *æqualia ſunt, quadrato* A F. *quod à tota* A B. *fit.*

Prob. Ex A B. fiat a quadratum E B. a 46. 1.
ex C. & D. erigantur b C G. D H. b 31. 1.
parallelæ & æquales ipſi A E. hoc & 3. 1.
poſito, erit rectangulum E C. comprehenſum ſub tota A E. c hoc eſt A B. & c 30.
ſegmento A C. & eodem modo rectan- Def.
gula G D. H B. ſub tota & utrolibet ſegmentorum. Cum ergo rectangula E C. G D. H B. ſint partes omnes ſuo toti quadrato A F. æquales, patet rectangula comprehenſa ſub A E. hoc eſt A B. & ſegmentis A C. C D. D B. æqualia eſſe quadrato lineæ A B. Q. E. D.

In numeris divide 10. in 7. & 3. dico 70. & 30. qui producuntur ex multiplicatione 10. in 7. & in 3. æqualia eſſe 100. quadrato numeri 10.

PROPOSITIO III.



Th. 3. *Si recta linea* AB. *ſecta ſit utcunque in* E. *Rectangulum* CB. *ſub tota* AB. *& uno ſegmentorum* AC. hoc eſt AE. *comprehenſum, æquale eſt rectangulo* FB. *quod ſub ſegmentis* BE. FE. hoc eſt BE. *comprehenditur, & quod à prædicto ſegmento* AE. *deſcribitur quadrato* CE.

a 11. 1.
b 31. 1. & 3. 1.
c 33. 1.

Prob. Datam AB. ſeco utcunque in E. ex punctis A. E. B. erigo [a] perpendiculares AC. EF. BD. parallelas [b] inter ſe & æquales ſegmento AE. tum duco rectam à puncto C. ad D. quæ erit parallela [c] ipſi AB. Hoc poſito ſic dico, AC. eſt æqualis [d] ipſi

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[d] ipſi AE. ergo rectangulum AD. eſt comprehenſum ſub tota AB. & uno ſegmentorum AC. hoc eſt AE. Rurſus FE. eſt [d] æqualis ipſi EA. ergo rectangulum FB. eſt comprehenſum ſub ſegmento BE. EF. hoc eſt AE. Denique parallelogrammum AF. quadratum eſt cum AC. EF. ſint perpendiculares & æquales ipſi AE. Ergo cum rectangulum AD. æquale ſit quadrato AF. & rectangulo FB. patet rectangulum ſub tota. AB. & ſegmento AE. æquale eſſe rectangulo comprehenſo ſub ſegmentis AE. EB. & quadrato prædicti ſegmenti AE. Q. E. D.

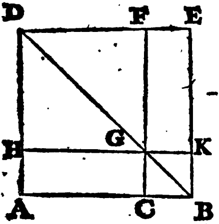
d Ex conſt.

In numeris divide 10. in 7. & 3. numerus 70. productus ex 10. in 7. æqualis eſt numero 21. qui ex 7. in 3. producitur; una cum 49. quadrato prioris partis 7.

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PROPOSITIO IV.

Tb. 4.



Si recta linea A B. *ſecta ſit utcunque*, in C. *quadratum* A E. *quod à tota* A B. *deſcribitur*, *æquale erit quadratis* H F. C K. *quæ à ſegmentis* A C. C B. *deſcribuntur*, *& ei rectangulo quod bis ſub ſegmentis* A C. C B. *comprehenditur* nempe rectangulis A G. G E.

a 46. 1. Prob. Super datam A B. fiat a quadratum A E. duc diametrum D B.
b 31. 1. ex C. fiat C F. parallela b recta B E. ſecans diametrum in G. per quod age H K. parallelam b ipſi A B. hoc poſito ſic dico. Trianguli A B D. latera A D. A B. ſunt æqualia. ergo anguli A D B.
d 5. 1. A B D. ſunt d æquales, ergo e ſemirecti,
e 32. 1. cùm angulus A. ſit rectus. Idemque
f 29. 1. dicendum de triangulo E D B. Rurſus angulus D F G. rectus f eſt, angulus F D G. oſtenſus eſt ſemirectus, ergo an-
g 32. 1. gulus F G D. etiam g ſemirectus eſt,
h 6. 1. ergo latera D F. F G. ſunt h æqualia:
i 34. 1. ſed ipſis etiam ſunt æqualia i latera op-
l 30. Def. poſita D H. H G. ergo parallelogrammum F H. quadratum l eſt. Eadem de cauſa

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cauſa quadratum erit C K. ergo H F. C K. quadrata ſunt ſegmentorum A C. C B. cùm latus H G. ſit æquale, ipſi A C. Similiter rectangula A G. G E. continentur ſub ſegmentis A C. C B. quia CG. GK. ſunt æquales ipſi CB. cum CK. ſit quadratum: ſic etiam G F. eſt æqualis rectæ H G. ob quadratum H F. hoc eſt rectæ A C. Igitur cum quadratum A E. ſit æquale quadratis H F. C K. & rectangulis AG. G E. verum eſt quadratum A E. ſuper datam A B. æquale eſſe quadratis ſegmentorum A C. C B. & rectangulo comprehenſo ſub iiſdem ſegmentis, bis ſumpto. Q. E. D.

Si dividatur 6. in 4. & 2. quadratum 6. hoc eſt 36. æquale eſt quadratis partium 4. & 2. hoc eſt 16. & 4. una cum rectangulo bis ſumpto ex numero 4. in 2. quod profert 8.

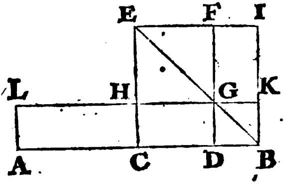
Coroll. 1. Hinc manifeſtum parallelogramma circa diametrum quadrati eſſe quadrata.

Coroll. 2. Diametrum quadrati dividere ejus angulos bifariam.

Coroll. 3. Si recta linea bifariam ſecetur quadratum totius lineæ æquari quatuor quadratis ex dimidia.

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PROPOSITIO V.



Tb. 5. *Si recta linea* A B. *ſecetur in æqualia in* C. *& non æqualia in* D. *Rectangulum* L D. *ſub inæqualibus totius* A D. *ſegmentis* A D. D G. hoc eſt D B. *comprehenſum, unà cum quadrato* H F. *ab intermedia ſectionum* C D. *æquale eſt quadrato* C I. *quod à dimidia* C B. *deſcribitur.*

Prob. Super dimidia C B. fiat, a 46.1. [a] quadratum C I. ductaque b 31.1. diametro B E. agatur [b] per D. recta D F. ipſi B I. parallela: Ex eadem recta B I. ſume BK. æqualem ipſi D B. & per punctum K. agatur K L. ipſi A B. parallela, ut.

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ut & AL. parallela ipſi BK. hoc poſito ſic dico. Rectangulum CG. [d] æquatur rectangulo GI. igitur addito communi [e] quadrato DK. erit CK. rectangulum æquale rectangulo DI. ſed AH. [f] æquatur rectangulo CK. ergo AH. [g] æquatur DI. ſi itaque addatur commune CG. erit rectangulum AG. æquale gnomoni IGC. quare cum gnomon IGC. cum quadrato [e] HF. intermediæ ſectionum æquatur quadrato CI. erit quoque rectangulum AG. cum prædicto quadrato HF. æquale quadrato CI. à dimidia. Q. E. D.

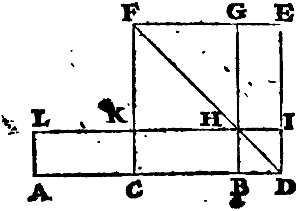
d 43.1.
e Corr. 2. præced.
f 36. 1.
g Ax. 1. 1.
e Corr. 2. præced.

Divide 10. æqualiter in 5. & 5. inæqualiter in 7. & 3. eritque numerus 21. ex 7. in 3. una cum quadrato numeri intermedii 2. quod eſt 4. æquale quadrato dimidii 5. hoc eſt numero 25.

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PROPOSITIO VI.



Th. 6. *Si recta linea* AB. *ſecetur biſariam in* C. *eique recta quædam* BD. *in rectum adjiciatur, rectangulum* AI. *comprehenſum ſub tota* AB. *cum adjecta* BD. *& ſub adjecta* DI. hoc eſt BD. *una cum quadrato* KG. *à dimidia* KH. hoc eſt CB. *æquale eſt quadrato* CE. *à linea* CD. *quæ tum ex dimidia* CB. *tum ex adjuncta* BD. *componitur tanquam una linea, deſcripto.*

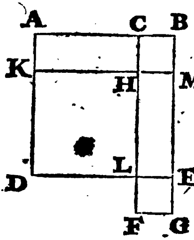
a 46.1. Prob. Super rectam CD. [a] fiat quadratum CE. per B. age b 31.1. BG. parallelam [b] ipſi DE. ſume DI. æqualem ipſi DB. & ex I. age IL. parallelam & æqualem ipſi DA. jungaturque recta LA. quo

quo facto ſic dico. Rectangula LC. KB. ſunt inter eaſdem paral- b 36.1.
lelas & ſupra æquales baſes, [b] ergo c 45.1.
æqualia. Eidem K B. [c] æquale eſt complementum H E. ergo erit & H E. æquale ipſi L C. & additis communibus C H. B I. gnomon G H K. æqualis erit toti rectangulo AI. quod continetur ſub tota A B. cum adjecta B D. & ſub adjecta D I. hoc eſt B D. Jam vero gnomon G H K. adjecto quadrato KG. partis dimidiæ KH. [d] hoc d 34.1.
eſt CB. eſt æqualis quadrato ipſius CD. compoſito ex dimidia cũ adjuncta. Ergo parallelogrammũ AI, adjecto eodem quadrato K G. fiet ęquale eidẽ quadrato CE. Q.E.D.

In numeris 10. ſecetur bifariam in 5. & 5. addatur ei numerus 2. numerus 24. qui producitur, ducto compoſito 12. in adjunctum 2. una cum quadrato 25. quadrato dimidii æqualis eſt 49. quadrato numeri 7. qui ex dimidio 5. & adjecto 2. componitur.

PRO-

PROPOSITIO VII.

Th. 7. 

Si recta linea AB. *ſecetur utcunque in* C. *quadrata totius & utriuſvis ſegmenti* CB. *ſimul ſumpta, hoc eſt* AE. EF. *æqualia ſunt bis ſumpto rectangulo* AM. *quod ſub tota* AB. *& ſub dicto ſegmento* CB. *continetur, cum addito* KL. *alterius ſegmenti* AC. *quadrato.*

a 46.1. Prob. Super AB. a fiat quadratum AE. ſume BM. æqualem ipſi CB. ducantur CL. MK. parallelæ ipſis BE. AB. produc BE. in G. ſic ut EG. ſit æqualis

c 2. Ax. ipſi BM. c hinc erit MG. æqualis ipſi BE. fiat quadratum EF. hoc poſito : quadratum totius AB. quod eſt AE. cum quadrato ſegmen-

ſegmenti C B. [d] hoc eſt E F. æqualia ſunt rectangulis A M. M F. (quæ ſunt ſub tota A B. & ſegmento B C. cum B M. ſit ipſi B C. æqualis; & in rectangulo M F. latera M G. F G. ſint æqualia ipſis B E. B M. hoc eſt A B. C B.) una cum quadrato alterius ſegmenti A C. quod eſt K L. totum videlicet partibus omnibus eſt æquale. Q. E. D.

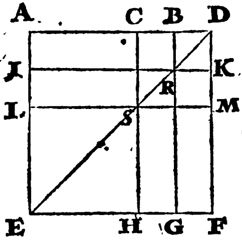
[d] Ex conſt.

Divide 6. in 4. & 2. quadratum totius 6. nempe 36. una cum quadrato ipſius 2. hoc eſt 4. æqualia ſunt numero 40. qui fit ex numero 6. bis ducto in 2. hoc eſt 24. una cum quadrato alterius partis 4. quod eſt 16.

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PROPOSITIO VIII.

Th. 8.



Si recta linea A B. *ſecetur utcunque* in C. *rectangulum quater comprehenſũ ſub tota* A B. *&* *uno ſegmentorum* B R. hoc eſt B C. *cum eo, quód à reliquo ſegmento* A C. id eſt L S. *fit*, *quadrato* L H. *æquale eſt quadrato* A F. *quod à tota* A B. *&* *dicto ſegmento* B D. hoc eſt B C. *tanquam ab una* A D. *deſcribitur*.

Prob. Rectæ A B. ſectæ in C. adjiciatur in rectum B D. ipſi B C. æqualis. Super tota A B. & adjuncta B D. hoc eſt ſuper A D. a 46.1. fiat[a] quadratum E D. ex punctis B. & C. duc rectas B G. C H. ipſi D F. parallelas, acceptiſque D K. K M. ipſis D B. B C. æqualibus, duc rectas K I. M L. ipſi D A. parallelas. Hoc poſito ſic dico, circa R. conſtituta ſunt quadrata quatuor, quorum latera omnia ipſi

ipſi B C. ſunt [a] æqualia. Ducta diametro E D. complementa AR. RF. [b] ſunt æqualia, ſuntque rectangula ſub toto A B. & B R. hoc eſt ſegmento B C. Eodemque modo I S. S G. ſunt complementa æqualia, quibus ſi addas quadrata æqualia S R. B K. fient rectangula duobus præcedentibus æqualia, cum ſint inter eaſdem parallelas & æquales baſes: ergo quatuor illa rectangula ſunt ſub tota & uno ſegmento. Quod ſi quatuor illis rectangulis addas quadratum LH. alterius ſegmenti L S. hoc eſt A C. illa omnia ſimul ſumpta erunt æqualia quadrato ED. quod fit ſupra AD. Q.E.D.

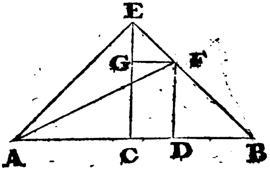
a Corr. 2. 4. hujus.

b 31. 1.

Si 6. ſecentur in 4. & 2. ducaturque quater numerus 6. in 2. fient 48. & addatur quadratum ipſius 4. hoc eſt 16. fiet numerus 64. æqualis quadrato ipſius 8. qui numerus componitur ex toto 6. & parte 2.

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PROPOSITIO IX.



Th. 9. *Si recta linea* A B. *ſecetur in æqualia* in C. *& non æquali- in* D. *quadrata quæ ab inæqualibus ſegmentis* A D. D B. *fiunt, dupla ſunt, eorum quæ à dimidia* A C. *& ab intermedia* C D. *fiunt.*

Prob. Ex C. erigatur C E. perpendicularis ipſi A B. & æqualis ipſi C A. vel C B. ducanturque rectæ E A. E B. Deinde ex D. erigatur D F. ipſi E C. parallela ſecans E B. in F. & fiat recta F G. ipſi C D. parallela, ducaturque recta A F. hoc poſito: Trianguli a Iſoſcelis A C E. anguli A. & E. ſunt b æquales c & ſemirecti, cum angulus A C E. ſit rectus. Idem dicendum de triangulo E C B. ergo totus angulus AEB. rectus eſt. Jam in triangulo E G F.

a *Ex conſt.*
b 5. 1.
c 32. 1.

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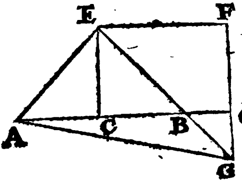
angu-

angulus G. d æqualis eſt angulo ECB. d 29.1.
a ergo rectus, ergo anguli E. & F. b æquales c quia angulus E. ſemirectus eſt: e ergo latera GE. GF. æqualia. Unde e 6. 1.
cum GF. æquatur f ipſi CD. erit quo- f 34. 1.
que GE. æqualis CD. Simili argumento probatur DF. æqualis ipſi DB.
Jam quadratum rectæ AF. g æquale g 47. 1.
eſt quadratis ſegmentorum inæqualium AD. DF. hoc eſt DB. Rurſus quadratum rectæ AF. g æquale eſt quadratis AE. EF. Eſt autem AE. æquale ipſis AC. CE. atque adeo duplum quadrati quod fit à dimidia AC. Et quadratum EF. æquale eſt quadratis EG. GF. atque adeo duplum quadrati quod fit à ſegmento medio GF. ſeu CD. quare quadrata quæ fiunt ab inæqualibus ſegmentis AD DB. dupla ſunt eorum quæ à dimidia AC. & ab intermedia ſectione fiunt. Q. E. D.

Divide 10. in 5. & 5. & in 7. & 3. media ſectio 2. quadrata 49. & 9. partium inæqualium 7. & 3. ſunt duplum quadratorum 25. & 4. & partis dimidiæ 5. & ſectionis 2.

PROPOSITIO X.

Th. 10.



Si recta AB. ſecetur bifariam in C. eique adjiciatur in directum recta B O. *quod à tota cum adjuncta* A O. *& quod ab adjuncta* B O. *utraque ſimul quadrata, dupla ſunt quadrati à dimidia* A C. *& ejus quod à compoſita ex dimidia* C B. *& adjuncta* B O. *tanquam una deſcribitur.*

Prob. Ex C. erigatur perpendicularis C E. æqualis ipſi A C. jungatur rectæ A E. E B. ex E. fiat EF. parallela ipſi CO. per O. ducatur O F. parallela ipſi C E. occurrens rectæ E B. in G. jungaturque recta AG. In triangulo A C E. latera A C. E C. ſunt æqualia, & angulus ad C. rectus: ergo reliqui ſemirecti: itidemque in triangulo E C B. Similiter in trian-

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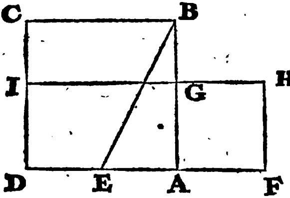
triangulis E F G. & BOG. latera E F. F G. ac BO. GO. ſunt [a] æqualia, quia ang. ad O. rectus & B. ſemirectus unde reliqui ſemirecti & æquales. [a] 6. 1.

Quare cum in triangulo AOG. angulus ad O. rectus eſt : erit quadratum rectæ A G. æquale [b] quadratis rectarum AO. & OG. [b] 47. 1. hoc eſt B O. rurſus in triangulo A E G. angulus ad E. rectus eſt conſtans ex duobus ſemirectis: ergo quadratum ipſius A G. æquale eſt quadratis A E. & E G. Eſt autem A E. duplum quadrati A C. & E G. duplum quadrati E F. vel F G. ergo etiam quadrata A O. & B O. dupla ſunt ipſorum A C. & C O. Q. E. D.

Numerus 10. ſecetur in 5. & 5. cui addantur 3. quadrati 169. & 9. numerorum 13. & 3. dupli ſunt numerorum quadratorum 25. & 64. qui ex numeris 5. & 8. gignuntur.

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PROPOSITIO XI.



Prob. 1. *Datam rectam* A B. *ita ſecare in* G. *ut rectangulum* C G. *comprehenſum ſub tota* A B. *& ſub uno ſegmentorum* G B. *ſit æquale alterius ſegmenti* A G. *quadrato* G F.

Praxis. Ad punctum A. excita perpendicularem A D. æqualem datæ A B. eam ſeca bifariam in E. duc rectam E B. & ipſi æqualem E F. producendo E A. Ex A B. abſcindo A G. æqualem A F. & factum erit quod quæritur.

Prob. Supra datam AB. perfice quadratum A C. & ſupra rectam A F. quadratum F G. & rectam H G. produc in I. hoc poſito ſic dico. Recta D A. [a] ſecta eſt bifariam

a Ex nſt.

fariam in E. eique in directum adjecta eſt A F. [b] ergo rectangulum F I. quod factum eſt ſub tota D F. & F H. hoc eſt F A. una cum quadrato mediæ E A. æquale eſt quadrato E F. hoc eſt E B. Jam quadratum E B. [c] æquale eſt quadratis A B. A E. ergo quadrata A B. A E. ſunt æqualia rectangulo F I. cum quadrato E A. Ergo ſi commune quadratum A E. tollas, rectangulum F I remanebit æquale quadrato ipſius A B. hoc eſt A C. Quod ſi ab æqualibus A C. F I. tollas commune A I. remanebit C G. rectangulum ſub tota C B. hoc eſt B A. & altero ſegmentorum G B. æquale quadrato G F. quod fit à reliqua parte G A. Q. E. D.

b 6. 2.

c 47. 1. Th. 11.

SCHOLIUM.

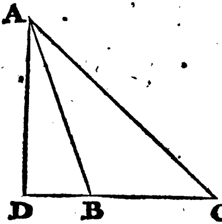
Hæc propoſitio numeris explicari nequit & idem denotat, quod tertia definitio libri ſexti de media ac extrema alicujus lineæ ſectione.

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PROPOSITIO XII.

Th. 11.



In amblygonio triangulo ABC. quadratum lateris AC. angulum B. obtuſũ ſubtendentis, quadrata laterum BA. BC. angulum obtuſum comprehendentium, ſuperat bis ſumpto rectangulo ſub latere BC. & ſub ipſa BD. in directum ei addita uſque ad occurſum perpendicularis ab A. altero angulo acuto cadentis.

Prob. Demitte perpendicularem ex A. & rectam CB. produc uſque dum ei occurrat in D. Quia recta CD. diviſa eſt
a 4.2. utcunque in B. [a] eſt quadratum ipſius CD. æquale quadratis rectarum DB. BC. cum duobus

rectan-

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rectangulis ſub DB. B C. addatur ergo utrimque quadratum rectæ D A. erunt quadrata C D. D A. æqualia tribus quadratis CB. BD. D A. cum duobus illis rectangulis, atqui quadratum rectæ A C. eſt æquale quadratis ipſarum CD. D A. & quadratum ipſius A B. eſt æquale quadratis ipſarum BD. D A. ergo quadratum rectæ A C. eſt æquale duobus quadratis C B. BA. cum duobus illis rectangulis. Superat ergo A C. duo quadrata duobus iſtis rectangulis ſub C B. in D B. Q. E. D. *per 47. 1.*

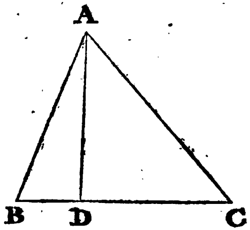
SCHOLIUM.

Hinc fluit generalis illa geometrarum regula ex tribus amblygonii trianguli lateribus ſegmentum D B. *inveniendi: nimirum ex quadrato* A C. *ſubt. ſumma quadratorum* A B. *&* B C. *reliquum diviſum per duplum baſeos* C B. *exhibebit ipſum* D B.

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PROPOSITIO XIII.



Th. 12. *In Oxygonio triangulo* A B C. *quadratum lateris* A B. *angulum* C. *acutum ſubtendentis ſuperatur à quadratis laterum* C A. C B. *eundem comprehendentium, bis ſumpto rectangulo ſub latere* C B. *& ſub aſſumpta interiùs linea* D C. *uſque ad occurſum perpendicularis ab* A. *altero angulo acuto cadentis.*

Prob. Demitte perpendicularem A D. Recta B C. diviſa eſt utcunque in D. ergo per 7. 2. quadrata rectarum B C. D C. æqualia ſunt rectangulis duobus ſub B C. C D. & quadrato reliqui ſegmen-

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ſegmenti B D. Adde utriſque commune quadratum rectæ D A. ſic tria quadrata B C. D C. D A. æqualia ſunt quadratis duobus B D. D A. & rectangulis duobus ſub B C. D C. Nunc quadratis duobus D C. D A. [a] æquale eſt quadratum A C. Ergo duo quadrata rectarum B C. C A. æqualia ſunt rectangulo bis ſumpto ſub B C. D C. & quadratis B D. D A. hoc eſt quadrato A B. Ergo quadratum rectæ B A. minus eſt quadratis A C. C B. rectangulo bis ſumpto ſub rectis B C. D C. Q. E. D.

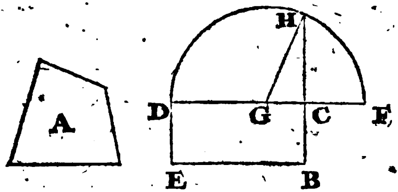
a 47. 1.

SCHOLIUM.

Hinc altera Generalis regula Geometrarum conſtat in triangulo acutangulo ex tribus lateribus invenire ſegmentum baſis, ſcil. adde quadr. A C. *ad quadr.* B C. *ſubtrahatur ex ſumma quadr.* A B. *reliquum dividatur per duplum baſeos* B C. *&* *proveniet* D C.

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PROPOSITIO XIV.



Th. 13. *Dato rectilineo* A. *æquale quadratum* C H. *conſtituere.*

PEr 45. 1. fiat rectangulum B D. æquale rectilineo A. ſi rectanguli latera ſint æqualia, erit quadratum quod petitur. Si inæqualia, producas unum, puta D C. in F. ſic ut C F. æqualis ſit ipſi C B. ſeca bifariam D F. in G. & centro G. ſpatio D G. duc circulum D H F. produc latus B C. in H. quadratum quod fit ex C H. erit æquale rectangulo C E.

Prob. Recta D F. ſecta eſt æqualiter in G. & non æqualiter in C. [a] ergo rectangulum CE. ſub inæqualibus ſegmentis D C. C B.

a 5. 2.

hoc

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hoc eſt CF. una cum quadrato ſegmenti medii G C. æqualia ſunt quadrato rectæ G F. [b] hoc eſt GH. ſed quadratum GH. [c] æquale eſt quadratis GC. CH. & conſequenter quadrata G C. C H. æqualia ſunt rectangulo C E. & quadrato G C. Ergo ſi tollas commune quadratum G C. remanebit quadratum rectæ C H. æquale rectangulo C E. hoc eſt rectilineo A. quod erat faciendum.

b 15. Def. 1.
c 47. 1.

MONITUM.

In ſuperioribus, frequenter adhibui numeros: cum tamen in demonſtrationibus geometricis ſæpe uſui eſſe non poſſint; quia irrationales & incommenſurabiles quantitates non explicant. Sed nota 1. Semper in omnibus præponi geometricas demonſtrationes. 2. Non recipi quidem debere numeros in demonſtrandis ir-

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rationalium aut incommenſurabilium quantitatum habitudinibus & affectionibus, quæ ſola quantitate continua cognoſcuntur: verum nemo negarit in demonſtrationibus quantitatis continuæ majoris lucis gratia, & explicandæ clarius propoſitionis, nos poſſe uti numeris, modo eos non accipiamus pro fundamento rationis. Unde robur ſuum non accipit demonſtratio à numeris, ſed lucem tantum. Et vero iis uſus eſt Archimedes propoſit. 2. de circuli dimenſione & poſt eum omnes paſſim geometræ.

NOTA.

Hujus libri ſelectæ propoſitiones ſunt 5. 6. 12. 13.

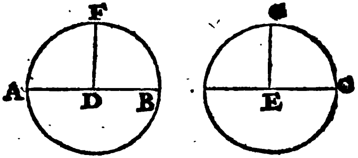
EUCLI-

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EVCLIDIS ELEMENTUM III.

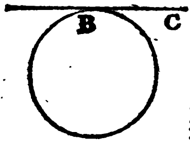
DEFINITIONES

I.



Æquales circuli ſunt, quorum diametri A B. B C. *ſunt æquales: vel quorum, quæ ex centris* D. & E. *recta lineæ* DF. EG. *ſunt æquales.*

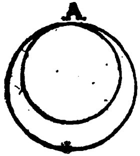
II.



Recta circulum tangere dicitur, quæ cum circulum tangat puta in B. *ſi producatur* in C. *circulum non ſecat.*

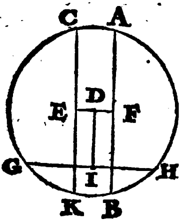
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III.



Circuli ſe mutuo tangere dicuntur qui ſeſe mutuo tangentes ut in A. *ſeſe mutuo non ſecant.*

IV.



In circulo æqualiter diſtare à centro recta dicuntur, cum perpendiculares DE. DF. *à centro* D. *ad ipſas* AB. CK. *ducta æquales ſunt; longius autem abeſſe dicitur* GH. *in quam major perpendicularis* DI. *cadit.*

V.

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V.



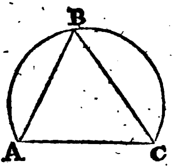
Segmentum circuli, eſt figura quæ ſub recta A B. *&* *circuli peripheria* A C B. *comprehenditur.*

VI.



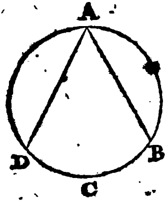
Segmenti autem angulus eſt C A B. *qui ſub recta linea* A B. *& circuli peripheria* C A. *comprehenditur.*

VII.



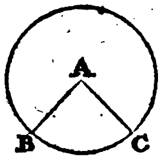
In ſegmento autem angulus eſt puta ABC. *cum in ſegmenti circumferentia ſumptum fuerit punctum quodpiam* B. *& ab eo in terminos rectæ* A C. *ſegmentum terminantes, lineæ rectæ ut* B A. B C. *fuerint ductæ.*

VIII.



Cum vero comprehendentes angulum D A B. *recta* A D. AB. *aliquam aſſumũt peripheriam ut* B C D. *illi angulus dicitur inſiſtere.*

IX.

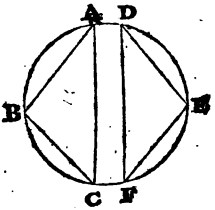


Sector circuli eſt, cum ad ipſius circuli centrum A. *angulus* B A C. *fuerit conſtitutus: comprehenſa nimirum figura & à rectis* A B. A C. *angulum* B A C. *continentibus, & à peripheria* B C. *ab illis aſſumpta.*

X.

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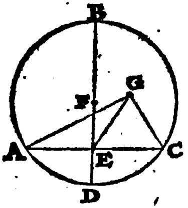
X.



Similia circuli ſegmenta ſunt ABC. DEF. *quæ angulos* B A C. E D F. *capiunt æquales, aut in quibus anguli* C B A. F E D. *inter ſe ſunt æquales.*

PRO-

PROPOSITIO I.



Prob. I. *Dati circuli* ABC. *centrum* F. *reperire.*

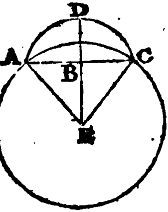
a 10. I. b 11. I. Prax. Ductam A C. a divide bifariam in E. Ad punctum E. b erige perpendicularem attingentem ambitum in B. & D. hanc B D. bifariam a ſeca in F. punctum F. erit centrum circuli.

c 15. I. Def. Prob. Non eſt aliud punctum in recta B D. c cum centrum ibi ſit tantum ubi linea ſecatur bifariam. Neque erit extra rectam B D. Sit enim in G. ducanturque G A. G E. G C. in triangulis G A E. d Ex conſt. G C E. Latera G A. A E. ſunt d æqualia ipſis G C. C E & G E. commune. Ergo e 8. I. tota triangula e ſunt æqualia, & anguli f 10. I. Def. G E A. G E C. æquales. f Ergo angulus G E A. rectus: quod eſſe non poteſt g Ex conſt. cum ejus partialis F E A. g ſit rectus.

Coroll. Si linea recta in circulo aliam lineam rectam bifariam & ad angulos ſecat, in ſecante erit centrum.

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PROPOSITIO II.



Si in peripheria circuli A B C. *duo qual. puncta* A. *&* C. *accepta fuerint, recta* AC. *quæ ad ipſa puncta adjungitur, intra circulum* A B C. *cadet.* Th. 1.

Prob. Si non cadat intra, cadat extra, ſitque recta A D C. Centro E. a reperto, ducantur rectæ E A. E C. ED. ſecetque E D. peripheriam in B. quia autem trianguli EADC. (qui rectilineus ab adverſario ponitur) latera E A. E C. ſunt b æqualia, erunt anguli c E A D C. ECDA. æquales. Eſt autem externus A D E. d major interno D C E. & per conſequens quam E A D. Ergo A E. & ei b æqualis E B. e major erit quam ED. pars quàm totum. Non ergo recta ex A. ad C. ducta, extra circulum cadet; ergo intra. Q. E. D.

a 1. 3.
b 15. Def.
c 5. 1.
d 16. 1.
e 19. 1.

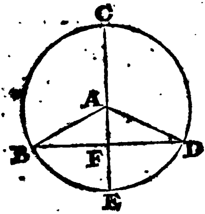
Coroll. Hinc patet lineam rectam circulum tangentem in uno tantum puncto tangere.

PRO-

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PROPOSITIO III.

Th. 2.



Si in circulo CBD. *recta quædam* CE. *per centrum* A. *rectam quadam* BD. *non per centrum, bifariam in* F. *ſecet, & ad* (angulos) *rectos eam ſecabit: Et ſi ad rectos eam ſecet, biſariam quoque eam ſecabit.*

Prob. 1. pars. Ductis à centro A. æqualibus rectis A B. AD. triangula A B F. A F D. habent omnia latera
a 8. 1. æqualia ſingula ſingulis: a ergo anguli
b 10. 1. AFB. AFD. ſunt æquales, b ergo recti.

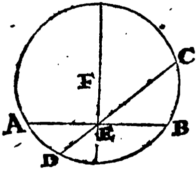
Prob. 2. pars. Latera A B. A D. ſunt
c 5. 1. æqualia: angulus A B D. c æqualis eſt
d *Ex conſt.* angulo A D B. & A F B. d ipſi A F D. Ergo latera e B F. F D. ſunt æqualia.
e 26. 1. Q. E. D.

Coroll. In omni triangulo ſeu æquilatero ſeu Iſoſcele linea recta baſin biſariam ſecans, ad eandem perpendicularis eſt & contra.

PRO-

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PROPOSITIO IV.



Si in circulo Th. 3. A D B. *duæ rectæ* A B. C D. *ſe invicem ſecent, non per centrum* F. *extenſæ, ſeſe biſariàm non ſecant.*

Prob. Si una tantum per centrum tranſeat & alia non: [a] ergo altera alteram non ſecabit bifariam. Si neutra tranſeat. Ex centro F. in punctum ſectionis E. duco rectam F E. & ſic dico. Si rectæ A B. C D. forent biſectæ in F. ang. FEB. & FEC. [b] forent recti & proinde æquales. Q. E. A.

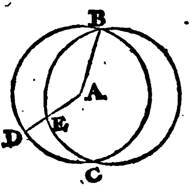
a 15. Def. 1.

b 3. 1.

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PROPOSITIO V.

Tb. 4.



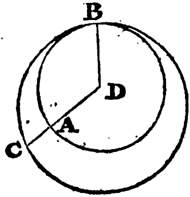
Si duo circuli D C B. E C B. *ſeſe mutuo ſecent* in B. & C. *non erit illorũ idem centrum* A.

Prob. Ductæ rectæ AB. AD. erunt æquales, cùm ſint à centro ad circumferentiam. Rectę etiam AE. AB. erunt æquales, ob eandem rationem ergo AE. erit [a] æqualis ipſi AD. Q. E. A.

a 1. *Ax.* 1.

PRO-

PROPOSITIO VI.



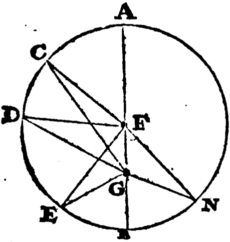
Si duo circuli Th. 5. AB. CB. *ſe ſe mutuo interius tangant* in B. *eorum non erit idem centrum* D.

Prob. Ductis D B. D C. linea D A. eſt æqualis lineæ D B. cùm ſint ductæ à centro ad circumferentiam. Lineæ DC. DB. ſunt æquales ob eandem cauſam. Ergo D A. D C. erunt æquales, pars toti, quod repugnat.

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PROPOSITIO VII.

Th. 6.



Si in circuli diametro AB. *ſumatur aliquod punctũ* G. *quod non ſit centrum circuli: & à puncto* G. *quadam recta* GC. GD. GE. GN. *in circulum cadant: maxima quidem erit* GA. *in qua centrum* F. *minima vero reliqua* GB. *aliarum vero ſemper ejus, qua per centrum ducitur, propior* GC. *remotiore* GD. *major erit: ſolum autem dua recta* GE. GN. *ab illo puncto* G. *aquales in circulum cadunt ad utraſque* (partes) *minima vel maxima.*

Prob. 1. pars. Ductis rectis FC. FD. FE. FN. ex centro F. duo latera CF. FG. trian-
a 20.1. guli CFG. ª majora ſunt tertio CG. at hæc ſunt æqualia toti GA.

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G A. ergo G A. eſt majus quam G C. Q. E. D.

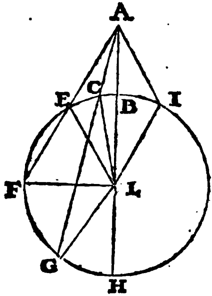
Prob. 2. Latera E G. G F. trianguli EGF. [a] majora ſunt tertio E F. ergo majora ſunt quàm linea F B. quæ eſt æqualis ipſi FE. ergo ſi dematur utrique communis recta G F. remanebit G E. major quam G B. Q. E. D. [a] 20.1.

Prob. 3. Triangula C F G. D F G. habent latera F C. F D. æqualia & latus F G. commune, angulus vero C F G. major eſt angulo DFG. totum parte: ergo latus CG. [b] majus erit quam DG. Q. E. D. [b] 24.1.

Prob. 4. Facto angulo GFN. æquali G F E. G N. G E. erunt [c] æquales. Nec à puncto G. aliæ duci poſſunt æquales ipſis G E. G N. erunt enim ſemper propiores ei quæ ducitur per centrum vel remotiores, & conſequenter majores vel minores, per tertiam partem hujus. Q. E. D. [c] 4.1.

PROPOSITIO VIII.

Th. 7.



Si extra circulum BEH, *ſumatur punctum quodpiam* A. *& à puncto ad circulum ducantur rectæ quædam* AF. AG. AH. *quarum una quidem per centrum* L. *reliqua verò ut libet. In cavam quidem peripheriam cadentium rectarum maxima* (erit) *quæ per centrum* L. (ducitur) *aliarum vero ſemper propior* (ei) *quæ per centrum* L. *remotiore major erit. In convexam vero peripheriam cadentium rectarum minima quidem eſt illa quæ inter punctum* A. *& diametrum* BH. (*ponitur*) *Aliarum vero ea quæ propior eſt minimæ* AB. *remotiore ſemper minor eſt. Duæ autem tantum rectæ æquales ab eo puncto* A. *cadent in circulum ad utraſque partes minimæ* AB. *vel maximæ* AH.

Prob. 1. pars. Ductis rectis LG. LF. duo latera AL. LG. hoc eſt LH. a majora ſunt tertio AG. ergo AH. major erit quam AG. Q. E. D.

a 20. 1.

Prob. 2.

Prob. 2. Latera A L. L G. trianguli A L G. ſunt æqualia lateribus L F. L A. trianguli A L F. angulus autem A L G. major eſt angulo A L F. b ergo latus A G. majus eſt latere A F. Q. E. D. b 24. 1.

Prob. 3. Ductis rectis L C. L E. duo latera A C. L C. trianguli A C L. a majora ſunt tertio A L. demantur æqualia L B. L C. remanebit A C. major quàm B A. Q. E. D. a 20. 1.

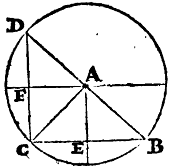
Prob. 4. Quia intra triangulum A L E. duæ rectæ A C. C L. junguntur: c erunt lateribus A E. E L. minores; demptis igitur æqualibus L C. L E. remanebit E A. major quam C A. Q. E. D. c 21. 1.

Prob. 5. Facto angulo A L I. æquali A L E. duo triangula illa d erunt æqualia: ergo latera A I. A E. æqualia; neque alia duci poteſt recta, his æqualis: erit enim ſemper propior minimæ A B. vel remotior & conſequenter e major vel minor per partem quartam hujus. Q. E. D. d 4. 1. e 21. 1.

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PROPOSITIO IX.

Tb. 8.



Si intra circulum BCD. ſumptum ſit aliquod punctum A. à puncto vero ad circulum cadant plures quam duæ rectæ æquales AB. AC. AD. acceptum punctum, centrum eſt circuli.

Prob. Ductis rectis BC. CD. diviſiſque bifariàm per rectas AE. AF. triangula ADF. ACF. [a] erunt æqualia: ergo anguli DFA. AFC. æquales: [b] ergo recti: ergo in linea FA. eſt circuli centrum. Rurſùs cum idem ſit de triangulis ACE. ABE. in rectâ AE. erit circuli centrum. Cum verò non ſit in duobus locis, debet eſſe ubi ſe interſecant. Q. E. D.

a 8. 1.
b 10. Def. 1.
c 1. 3.

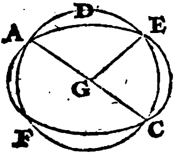
ALITER.

A nullo puncto plures quam, duæ rectæ ad circumferentiam duci poſſunt per 7. 3: ergo A. *erit centrum.* Q. E. D.

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PROPOSITIO X.



Circulus AEF. *non ſecat circulum* FDC. *per plura puncta quam duo.* *Th.* 9.

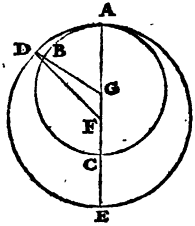
Prob. Secet enim in tribus ſi vis. Circuli EFC. centro G. [a] invento, ducantur rectæ GA. GC. GE. quæ, quia ſunt æquales, & attingunt ambitum circuli utriuſque, punctum G. [b] erit etiam centrum circuli utriuſque; quod eſt abſurdum per 5. hujus.

a 1. 3.
b 9. 3.

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PROPOSITIO XI.

Th. 10.



Si duo circuli ABC. AED. *contingant ſeſe interius* in A. *& ſumpta fuerint eorum centra* G. *&* F. *ad eorum centra adjuncta recta linea* F A. *& producta, in contactum* A. *cadet circulorum.*

Prob. Recta F G. conjungens eorum centra, non incidat in contactum ſed alibi in D. à puncto G. centro circuli A B C. ducatur recta G A. ad contactum ut & FD. latera G D. G F. [a] majora ſunt tertio F D. ergo majora [b] latere F A. dempto ergo communi F G. remanebit G A. majus latere GD. Eſt autem G A. æqualis lateri GB. ergo GB. majus erit quam G D. pars toto. Q. E. A.

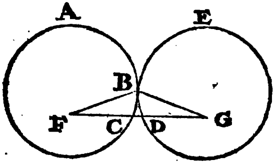
a 20. 1.

b 15. Def.

PRO-

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PROPOSITIO XII.



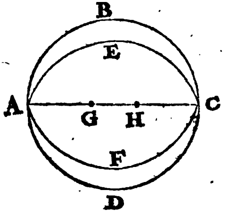
Si duo circuli A B C. E B D. *Th. 11.*
contingunt ſe invicem exterius in B.
quæ adjungitur ad eorum centra,
per contactum tranſibit.

Prob. Si neges: ſit recta F G. centra conjungens. Ductis F B. G B. latera B F. B G. [a] majora ſunt tertio F G. ſed BF. BG. ſunt æqualia radiis F C. G D. ergo quoque F C. G D. majora ſunt F C. C D. G D. pars toti. Q. E. A.

a 20.1.

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PROPOSITIO XIII.



Th. 12. *Circulus circulum non tangit in pluribus punctis, quam uno, ſive intus, ſive extra tangit.*

Prob. Tangat enim in duobus, puta A. & C. centrum [a] debebit eſſe in linea, quæ junget contactum circulorum: utriuſque autem non [b] poteſt eſſe idem centrum. Ergo in illa recta erunt duo centra, puta G. & H. quod fieri non poteſt, cum linea in unico puncto, poſſit tantum ſecari bifariàm.

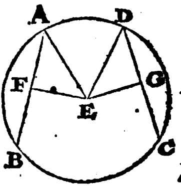
a 11. & 12. 3.

b 6. 3.

PRO-

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PROPOSITIO XIV.



In circulo A B C. *æquales rectæ* A B. D C. *æqualiter diſtant à centro* E. *& æqualiter diſtantes à centro, ſunt ſibi invicem æquales.* Tb. 13.

Prob. A centro E. in rectas AB. CD. duca perpendiculares EF. EG. rectæ AB. CD. ſectæ b erunt bifariam. Junctis EA. ED. quadratum rectæ ED. ceſt æquale quadratis rectarum DG. GE. ut & quadratum A E. quadratis rectarum A F. F E Demptis ergo ab æqualibus A F. F E. ipſis D G. G E. æqualium linearum quadratis AF. DG. remanebit recta E F. æqualis rectæ E G. & conſequenter rectæ A B. CD. d æqualiter diſtant à centro.

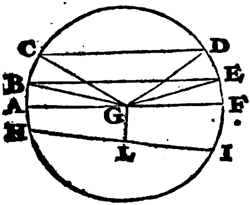
a 12. 1.
b 3. 3.
c 47. 1.
d 4. Def. 3.

Prob. 2. pars. Ex probatis quadrata EG. GD. ſunt æqualia quadratis EF. FA. & quadratum EG. æquale quadrato EF. ergo quadratum FA. æquale eſt quadrato GD. e ergo recta BA. æqualis eſt rectæ DC. Q. E. D.

e 7. Ax. 1.

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PROPOSITIO XV.



Th. 14. *In circulo* ABCD. *maxima quidem eſt diameter* AF. *aliarum verò ſemper propior* BE. *centro* G. *erit major remotiore* CD.

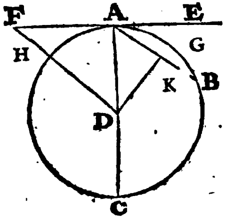
Prob. 1. pars. Ductis GB. GE. duo latera GB. GE. trianguli
a 20. 1. GBE. [a] majora ſunt tertio BE. at hæc ſunt æqualia diametro AF. ergo AF. major eſt quam BE. Q. E. D.

Prob. 2. Ductis rectis GC. GD. duo latera GC. GD. ſunt æqualia lateribus GB. GE. angulus vero BGE. major eſt angulo CGD.
b 24. 1. [b] ergo latus BE. majus latere CD. Q. E. D.

PRO-

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PROPOSITIO XVI.



Quæ ab extremitate diametri A C. *ad rectos angulos linea* E F. *ducitur, cadet extra circulum.* A B C. *& in locum inter ipſam* E F. *& circumferentiam,* ABC. *altera recta* AB. *non cadet: & ſemicirculi angulus* DAGB. *major erit omni acuto angulo rectilineo: reliquus autem* EAGB. *minor.* Th. 15.

Prob. 1. pars. Ex centro D. ducatur recta D H F. utcunque: latus D F. ſubtendens angulum F A D. rectum [a] majus erit D A. a 19. 1. hoc eſt D H. cum itaque H. ſit in circumferentia erit F. extra. Simili ratione de omnibus pūctis in linea FAE. argumentari licet. Q.E.D.

Prob. 2. pars. Ad AB. quæ inter peripheriam & rectam EF. caderet ducatur perpendicularis DK. ergo latus DA. majus erit
b 19. 1. [b] ipſi DK. ſed punctum A. eſt in circumferentia itaque K. & tota AB. erit intra circulum. Q. E. D.

Prob. 3. Ut fieret angulus major angulo DAGB. ſemicirculi, deberet duci recta inter rectam EA.& peripheriam AB.quod jam probavi fieri non poſſe.

Prob. 4. Si enim aliquis angulus rectilineus conſtitui poſſet minor angulo EAGB. contactus, duceretur recta inter AE. & peripheriam AB. quod, ut jam dixi, fieri non poteſt.

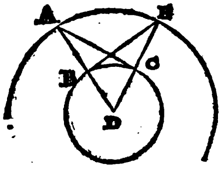
Corollarium.

Hinc communiter elicitur rectã ad extremum diametri perpendicularem, tangere circulum, & in unico puncto geometrice tangere:
c 2. 3. nam ſi plura tangeret, caderet [c] intra circulum.

PRO-

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PROPOSITIO XVII.



A dato puncto A. rectam lineam A C. *ducere, quæ datum tangat circulum* B C D. Prob. 2.

Praxis. Centro D. ſpatio A. fiat pars circuli A E. ducatur recta D A. & ad punctum B. excitetur perpendicularis B E. jungaturque recta D E. à puncto A. ducatur recta A C. hanc dico tangere circulum B C D.

Prob. Triangula ADC. BED. ſe habent juxta 4. 1. cum latera DA. DE. DB. DC. ſint [a] æqualia & angulus D. communis. Ergo cum angulus E B D. ſit rectus, rectus etiam erit D C A. recta itaque A C. [b] tanget circulum. Q. E. F.

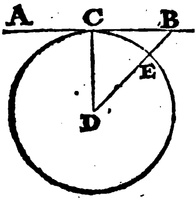
a 15. 1. Def.

b 16. 3.

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PROPOSITIO XVIII.

Tb. 16.



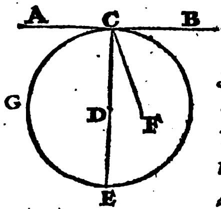
Si aliqua recta A B. *tangat circulum* D C E. *à centro vero* D. *ad contactum* C. *quadam recta* D C. *adjungatur: adjuncta* D C. *perpendicularis erit ad* A B. *quæ continget.*

Prob. Si negas: ſit alia, puta D B. perpendicularis, ergo cum angulus B. ponatur rectus erit angulus C.[a] minor recto, ergo latus D C. hoc eſt D E. [b] majus erit latere D B. pars toto quod eſt abſurdum.

a 17. 1.
b 19. 1.

PRO-

PROPOSITIO XIX.

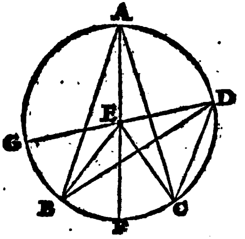


Si circulum E G C. *contingat aliqua recta* A B. *à contactu vero* C. *tangenti* AB. *ad rectos angulos recta linea* E C. *ducta ſit, in recta ducta* E C. *erit centrum circuli.* Th. 17.

Prob. Si negas, ſit alibi nimirum in F. proinde ducta F C. ipſi A B. [a] erit perpendicularis: ergo angulus rectus F C B. recto D C B. erit æqualis, pars toti quod eſt abſurdum. a 18. 3.

PROPOSITIO XX.

Th. 18.



In circulo DFGA. *angulus* BEC. *ad centrum* E. *duplũ eſt anguli* BAC. *ad peripheriam, cum fuerit eadem peripheria* BC. *baſis angulorum.*

Prob. Id tribus poteſt modis contingere. Includant 1. rectę AB. AC. rectas EB. EC. ductaque AF. per centrum E. duo latera EA. EB. erunt æqualia [a] ergo anguli EBA. EAB. æquales: angulus autem BEF. duobus EAB. EBA. [b] eſt æqualis, ergo duplus anguli BAF. Idem dic de angulo FEC. reſpectu anguli EAC. ergo totus BEC. totius BAC. erit duplus. Q.E.D.

a 5. 1.

b 32. 1.

2. Rectæ

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2. Rectæ D G. D B. non includant rectas E C. E B. iterum cum latera E D. E B. ſint æqualia erunt E D B. E B D. [c] anguli æquales. His autem duobus, angulus G E B. eſt [d] æqualis. Ergo idem erit duplus anguli G D B. Q. E. D.

c 5. 1.

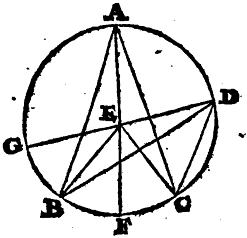
d 32. 1.

3. Triangula B E C. B D C. ſeſe interſecent, ducaturque recta D G. per centrum E. totus angulus G E C. erit duplus totius G D C. angulus vero G E B. duplus eſt anguli G D B. ergo reliquum B E C. duplum erit reliqui B D C. Q. E. D.

PRO-

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PROPOSITIO XXI.



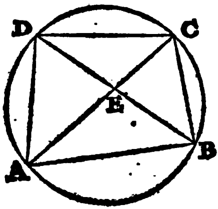
Tb. 19. *In circulo* A D C B. *qui in eodem ſegmento* B C. *ſunt anguli* B A C. B D C. *ſunt inter ſe æquales.*

a 20. 3. Prob. Angulus B E C. [a] eſt duplus anguli B A C. & duplus anguli B D C. [b] ergo anguli B A C. B D C. ſunt inter ſe æquales. Q. E. D.

b 1. *Ax.*

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PROPOSITIO XXII.



Quadrilatirorum in circulo ABCD. *deſcriptorum oppoſiti anguli* DCB. DAB. *duobus rectis ſunt æquales.* Th. 20.

Prob. Diametris AC. DB. ductis, anguli ADB. ACB. in eadem portione [a] ſunt æquales, ſimiliterque anguli BAC. BDC. ergo totus angulus ADC. eſt æqualis angulis BCA. BAC. ſed anguli BCA. BAC. cum tertio ABC. [b] valent duos rectos: ergo angulus ADC. æqualis ipſis BCA. BAC. cum angulo ABC. valebit duos rectos. Idem de aliis oppoſitis dicetur. Ergo, &c. Q. E. D.

a 21. 3.

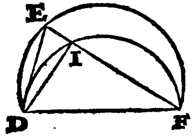
b 32. 1.

PRO-

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PROPOSITIO XXIII.

Tb. 21.



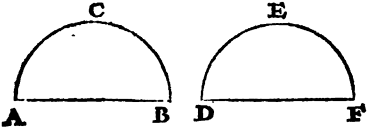
Super eadem recta DF. *duo ſegmenta circulorum ſimilia* DIF. DEF. *& inæqualia non conſtituentur ad eaſdem partes.*

Prob. Sint enim ſi fieri poteſt DIF. DEF. ſimilia ſegmenta, ductis rectis ED. EF. ID. anguli DIF. DEF. [a] erunt æquales, quod eſt abſurdum per 16. 1.

a 10. Def. 3.

PRO-

PROPOSITIO XXIV.



Super æqualibus rectis AB. DF. *ſimilia ſegmenta circulorum ſunt inter ſe æqualia.* Th. 22.

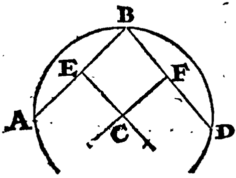
Prob. Collocetur AB. ſuper DF. [a] congruent. Etenim ſi ſegmenta non congruant vel unum totum extra aliud cadet, quod eſt abſurdum per 23. 3. vel cadet partim intra, partim extra; & ſic circulus circulum ſecabit in pluribus punctis quam duobus, quod repugnat per 10. 3.

a 8. *Ax.*

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PROPOSITIO XXV.

Prob. 3.



Circuli ſegmento dato A B D. *deſcribere circulum, cujus eſt ſegmentũ.*

Prax. Accipiantur in dato ſegmento tria puncta A B D. ductis rectis A B. B D. [a] diviſiſque bifariam & ad angulos rectos per rectas CE. CF. ſe mutuo interſecantes in puncto C. illud erit centrum.

a 10. & 11. 1.

Prob. Per 1. 3. centrum eſt in utraque C E. & C F. ergo ubi ſe interſecant. Circuli enim unius, unicum tantum poteſt eſſe centrum. Q. E. D.

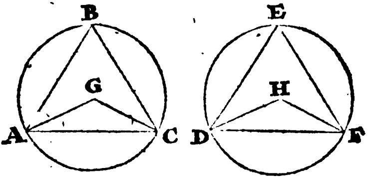
SCHOLIUM.

Hinc datis tribus punctis facile centrum circuli reperitur per data puncta tranſeuntis.

PRO-

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PROPOSITIO XXVI.



In æqualibus circulis ABC. DEF. *æquales anguli* G. *&* H. B. *&* E. *æqualibus peripheriis* AC. DF. *inſiſtunt, ſive ad centra* G. *&* H. *ſive ad peripharias* B. *&* E. *conſtituti ſint.* *Th.* 13.

Prima pars. Prob. Trianguli A G C. latera G A. G C. & angulus G. ponuntur æqualia lateribus H D. H F. & angulo H. a ergo baſes A C. D F. ſunt æquales. Ergo b peripheriæ A C. D F. erunt etiam æquales. Q. E. D.

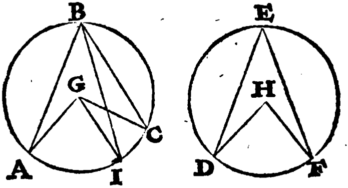
a 4. 1.
b 24. 3.

Prob. 2. Anguli A B C. D E F. ponuntur æquales: c ergo ſegmenta A B C. D E F. ſunt ſimilia: d ergo æqualia cum rectæ A C. DF. ſint æquales. Ergo cum circuli ponantur æquales, remanebunt ſegmenta A C. D F. e æqualia.

c Def. 10. 3.
d 23. 5.
e 3. Ax.

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PROPOSITIO XXVII.



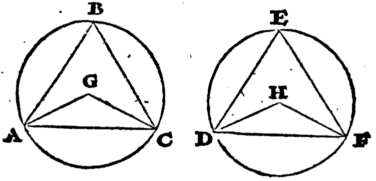
Th. 24. *In æqualibus circulis* ABI. DEF. *anguli qui in æqualibus peripheriis* AI. DF. *inſiſtunt ſunt inter ſe æquales, ſive ad centra* G. *&* H. *ſive ad peripherias* B. *&* E. *conſtituti, inſiſtant.*

Prob. Si non ſint æquales, ſit
a 23. 1. alter minor, puta AGI. [a] fiatque AGC. ipſi DHF. æqualis:
b 25. 3. ergo peripheria AC. erit [b] æqualis peripheriæ DF. ſed peripheria DF. ponitur æqualis ipſi AI.
c 7. ergo AC. & AI. erunt æquales,
Ax. pars toti: Idem [c] dic de angulis
d 20. 3. B. & E. cum G. & H. [d] ſint eorum dupli.

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PRO-

PROPOSITIO XXVIII.

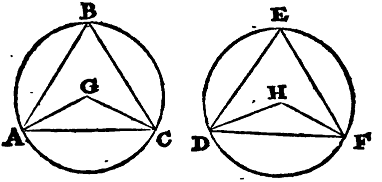


In æqualibus circulis ABC.DEF. *Th. 23.*
æquales rectæ AC. DF. *æquales*
peripherias AC.DF. ABC.DEF.
auferunt, majorem quidem majori;
minorem autem minori.

Prob. Ductis rectis GA.GC.
HD. HF. triangula AGC.
DHF. [a] ſunt æqualia. Ergo an- a 8. 1.
gulus G. angulo H. eſt æqualis:
ergo peripheriæ AC. DF. [b] æ- b 26.3.
quales. [c] ergo reliquæ ABC. c 3.
DEF. ſunt æquales. Q.E.D. Ax.

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PROPOSITIO XXIX.

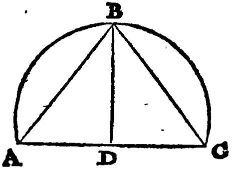


Th. 26. *In æqualibus circulis* ABC.DEF. *æquales peripherias* ABC.DEF. *æquales recta* AC. DF. *ſubtendunt.*

Prob. Ductis rectis GA.GC. HD. HF. anguli G. & H. a 27. 3. [a] erunt æquales: latera etiam GA. GC. HD. HF. ſunt æqualia ex ſuppoſitione: ergo baſes AC. b 4. 1. DF. [b] erunt æquales. Q. E. D.

PRO-

PROPOSITIO XXX.



Datam peripheriam A B C. *ſe-* *Prob. 4.*
care biſariam.

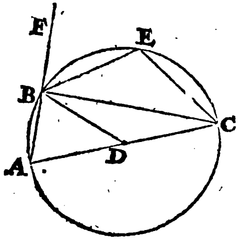
Praxis. Ducatur recta A C. quam divide [a] biſariam in D. a 10. 1.
per perpendicularem D B. erit peripheria ſecta biſariam in B.

Prob. Ductis rectis AB. CB. triangula A B D. D B C. ſe habent juxta 4. 1. ergo latera A B. C B. ſunt æqualia. [b] Ergo peri- b 28. 2.
pheriæ quas ſubtendunt ſunt æquales. Q. E. D.

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PROPOSITIO XXXI.

Th. 27.



In circulo ABEC. *angulus* ABC. *in ſemicirculo rectus eſt: qui autem in majore ſegmento* BAC. *minor recto: qui vero in minore ſegmento* BEC. *major recto: & inſuper angulus* CBA. ex recta CB. & peripheria BA. *majoris ſegmenti, recto quidem major eſt; minoris autem ſegmenti angulus* EBC. qui ex peripheria EB. & recta BC. *minor eſt recto.*

Prob. 1. pars. Centro D. ductis rectis DA. DB. DC. anguli

a 5. 1. DAB. DBA. [a] erunt æquales: itemque anguli DCB. DBC. ergo totalis angulus ABC. eſt æqualis angulis A. & DCB. ſed

his

his [b] eſt æqualis F B C. ergo angulus A B C. [c] eſt rectus. b 32. 1. c 13. 1.

Prob. 2. Angulus A B C. eſt rectus: ergo angulus B A C. in majore ſegmento [d] eſt minor recto. d 32. 1.

Prob. 3. Fiat quadrilaterum A B E C. angulus A. [e] minor eſt recto, ergo angulus BEC. in minori ſegmento [f] eſt major recto. e per 1. partem hujus. f 22. 3.

Prob. 4. Angulus ex peripheria A B. & rectæ C B. eſt major angulo recto compoſito ex rectis A B. B C. totum videlicet parte.

Prob. 5. Angulus compoſitus ex peripheria E B. & recta C B. minor eſt angulo F B C. recto compoſito ex recta FB. BC. pars toto. Hujus propoſitionis autor fertur Thales Mileſius annis ante Chriſtum, 650.

SCHOLIUM.

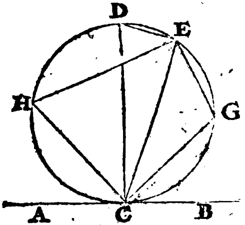
Hinc in triangulo rectangulo, ſecta hypothenuſa bifariam, erit illud punctum centrum circuli tria puncta illa pertranſeuntis, adeoque examen exacta normæ.

PRO-

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PROPOSITIO XXXII.

Tab. 28.



Si circulū CHEG. *tetigerit aliqua recta* AB. *à tactu autem* C. *ducatur quædam recta, ſecans circulum* D C. vel E C. *anguli quos ad tangentem* AB. *faciet, erunt æquales angulis qui ſunt in alternis circuli portionibus*, id eſt angulus A C E. æqualis eſt angulo G. & angulus B C E. angulo H.

Prob. Ducta perpendiculari D C. cum angulus A C D. ſit rectus, angulus qui fieret in
a 31:1. ſemicirculo, illi [a] eſſet æqualis: ſi vero non ſit rectus ut A C E. primo duc rectam D C. per centrum, deinde accipe in periphe-

ria

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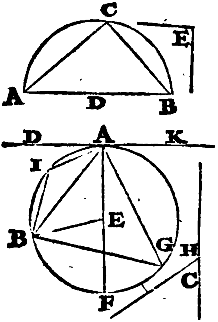
ria aliquod punctum puta G ducanturque rectæ D E. E G. G C. cum angulus D E C. in ſemicirculo [b] ſit rectus, reliqui duo puta E C D. E D C. [c] valent unum rectum: ſed anguli B C E. & E C D. valent etiam unum rectum, cum recta D C. ſit perpendicularis: dempto igitur communi E C D. remanebit B C E. æqualis angulo E D C. qui [d] æqualis eſt angulo C H E. ergo & angulus B C E. angulo C H E. æqualis. Rurſus, cum quadrilateri D G. anguli in circulo oppoſiti E D C. E G C. [e] valeant duos rectos, ſicut & anguli [f] A C E. E C B. & angulus C D E. ſit [g] æqualis angulo B C E. remanebit angulus G. angulo A C E. æqualis. Q. E. D.

b 13. 3.
c 32. 1.
d 27. 3.
e 22. 3.
f 13. 1.
g per 1. partem hujus.

PRO-

PROPOSITIO XXXIII.

Prob. 5.



Super data recta AB. *portionem circuli deſcribere, quæ capiat angulum dato angulo rectilineo æqualem.*

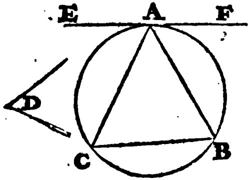
Si datus angulus ſit rectus, qualis eſt E. recta A B. diviſa bifariam in D. centro D. ſpatio, D A. ſi fiat ſemicirculus A C B. ductis rectis A C. C B. angulus
a 31. 1. C. [a] erit æqualis dato angulo E. quia erit in ſemicirculo. Si angulus ſit acutus ut C. ſitque data recta B A. ad punctum A. fiat an-
b 23. 1. gulut D A B. [b] æqualis angulo C. ductaque ad punctum A. perpendiculari F A. fiat angulus E B A.

æqua-

æqualis angulo E A B. latera EB. EA.[c] erunt æqualia: quare ſi puncto E. ſpatio E A. fiat circulus, tranſibit per punctum B. quo poſito ſic pergo. Cum recta F A. ſit diameter, & recta D A. ad ejus extremum ſit ei perpendicularis, [d] tanget circulum: ergo angulus D A B. [e] erit angulo cuicunque, qui fiet in alterna circuli portione, puta angulo AGB. æqualis: ergo portio A H G B. continet angulum æqualem angulo dato C. Si vero angulus ſit obtuſus puta H. eadem erit demonſtratio: angulus enim A I B. ipſi H. [f] erit æqualis.

c 6. 1.
d *per corol.* 16. 3.
e 32. 3.
f 22. 3.

PROPOSITIO XXXIV.



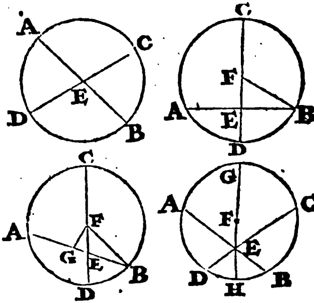
A dato circulo A B C. *ſegmentum* C B A. *abſcindere capiens angulũ* B. *æqualem dato angulo rectilineo* D.

[a] Ducatur tangens E F. ad punctum A. [b] fiat angulus CAE. æqualis dato D. portio ABC. [c] capiet angulum B. æqualem dato. Q. E. F.

a 17. 3.
b 23. 1.
c 32. 3.

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PROPOSITIO XXXV.



Th. 29. *Si in circulo* ADBC. *duæ rectæ* AB.CD. *ſe mutuo* in E. *ſecuerint, rectangulum comprehenſum ſub ſegmentis unius* A E. E B. *æquale eſt ei quod ſub ſegmentis alterius* CE. ED. *comprehenditur rectangulo.*

Prob. 1. Rectæ AB. CD. ſecent ſe in centro E. rectangulum unum, alteri erit æquale: cum omnes radii ſint æquales.

2. Sola CD. tranſeat per centrum F.
a 3. 3. dividatque rectam AB. bifariam in E. a ac proinde ad angulos rectos, ducaturque recta F B. quo facto, cum recta C D. ſecetur in æqualia in F. & non æqualia in E. erit rectangulum ſub inæqualibus ſegmentis C E. E D. cum quadrato ſeg-
b 5. 2. menti intermedii EF. b æquale quadrato dimi-

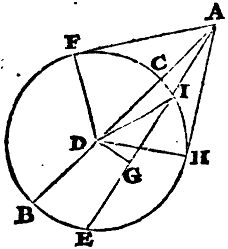
dimidiæ FD. vel FB. ſed quadratum FB. eſt c æquale quadratis BE. EF. quæ per conſequens æqualia ſunt rectangulo CE. ED. cum quadrato EF. Dempto igitur communi FE. remanebit rectangulum CE. ED. æquale rectangulo ſub BE. EA. Q. E. D. c 47. 1.

3. Recta CD. tranſiens per centrum F. rectam AB. non dividat bifariam in E. ductaque recta FB. & perpendiculari FG. rectangulum ſub CE. ED. cum quadrato FE. d erit æquale quadrato FD. vel FB. rectangulum etiam ſub AE. EB. cum quadrato GE. eſt æquale quadrato GB. adde quadratum FG. jam cum quadratum FB. ſit æquale quadratis FG. GB. erit rectangulum AE. EB. cum quadratis EG. GF. æquale quadrato FB. hoc eſt rectangulo CE. ED. & quadrato FE. ergo cum quadratum FE. ſit æquale quadratis FG. GE. ſi ab uno demas FE. & ab alio EG. GF. remanebunt æqualia rectangula CE.ED. & AE. EB. Q. E. D. d 5. 2.

4. Si neutra tranſeat per centrum & ſe ſecent utcunque, ducatur ad interſectionem E. recta GH. tranſiens per centrum: cum rectangulum ſub CE. ED. e ſit æquale ei quod ſub HE. EG. Idemque AE. EB. ſit æquale ipſi GE. EH. erunt æqualia rectangula ſub CE. ED. & AE. EB. Q. E. D. e *per 3. partem hujus.*

PROPOSITIO XXXVI.

Tb. 10.



Si extra circulũ FBE. *ſumatur punctum aliquod* A. *ab eoque in circulum cadant duæ rectæ: & hæc quidem* A B. *ſecet circulum* in C. *illa autem* A F. *tangat* in F. *Quod ſub tota ſecante* A B. *& exterius aſſumpta* A C. *inter punctum* A. *& convexam peripheriam* C. *comprehenditur rectangulum, æquale erit ei, quod à tangente* A F. *deſcribitur quadrato.*

Prob. Tranſeat 1. recta A B. per centrum D. ductaque recta D F. cum recta C B. bifariam ſecta ſit in D, & ei recta A C. adjiciatur, rectangulum ſub A B. & A C. contentum, una cum quadrato D C. vel D F. a æquale eſt ei quod à D C. cum A C. tanquam una linea fit quadrato. Sed quadratum D A. b eſt æquale quadratis D F. F A. ergo dempto communi F D. remanebit quadratum F A. æquale rectangulo ſub AB. & CA. Q. E. D.

a 6. 2.

b 47. 1.

2. Si

2. Si recta AE. non tranſeat per centrum, à centro D. duc perpendicularem DG. c hæc ſecabit rectam EI. biſariam, cum igitur recta EI. ſit ſecta bifariam in G. & ei recta IA. adjiciatur, erit rectangulum ſub AE. & ſub AI. cum quadrato GI. d æquale quadrato GA. addito ergo quadrato DG. erit rectangulum ſub AE. & ſub IA. cum quadratis IG. GD. hoc eſt quadrato DI. hoc eſt DF. æquale quadrato DA. ſed DA. eſt æquale quadratis FA. FD. demptis ergo æqualibus DF. DI. remanebit quadratum FA. æquale rectangulo ſub AE. & AI. Q. E. D.

c 3. 3.

d 6. 2.

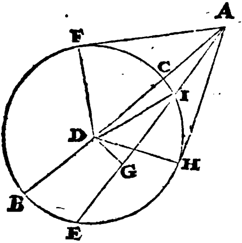
Coroll. 1. Hinc ſequitur, ſi à puncto quovis extra circulum ſumpto, plures rectæ circulum ſecantes ducantur, rectangula comprehenſa ſub totis lineis & partibus exterioribus, inter ſe eſſe æqualia.

Coroll. 2. Duæ rectæ, ab eodem puncto ductæ, quæ circulum tangunt, ſunt inter ſe æquales.

Coroll. 3. Ab eodem puncto extra circulum ſumpto, duci tantum poſſunt duæ rectæ, quæ circulum tangunt.

PROPOSITIO XXXVII.

Th. 31.



Si extra circulum F H E. *ſumatur punctum aliquod* A. *ab eoque puncto in circulum cadant duæ rectæ* AF. AB. vel AE. *& hæc quidem* AB. *ſecet circulum: illa autem* AF. *incidat: ſit autem quod ſub tota ſecante* AB. *& exterius aſſumpta* CA. *inter punctum & convexam peripheriam, rectangulum æquale ei quod ab incidente* AF. *deſcribitur: incidens illa circulum tanget.*

a 17. 3. Prob. a Duc tangentem AH. & ad H. rectam DH. cum

b 36. 3. ergo quadratum AH. b ſit æquale rectangulo ſub AB. CA. & idem rectangulum ſub AB. CA. ponatur

natur æquale quadrato F A. lineæ F A. H A. erunt æquales, latera item F D. H D. ſunt æqualia & baſis A D. communis: ergo tota triangula [c] ſunt æqualia. Ergo cùm angulus A H D. ſit [d] rectus, rectus etiam erit AFD. ergo AF. circulum tanget per coroll. 16. 3.

c 8. 1.
d 18. 3.

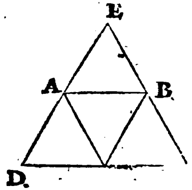
NOTA.

Selectiores hujus libri propoſitiones ſunt. 20. 22. 31. 35. 36.

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EVCLIDIS ELEMENTUM IV.

DEFINITIONES.



1. *Figura rectilinea, in figura rectilinea inſcribi dicitur, cum ſinguli, ejus figuræ, quæ inſcribitur, anguli, ſingula latera ejus quæ inſcribitur tangunt.*

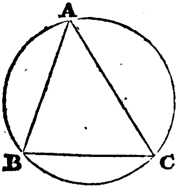
Ut triangulum A B C. inſcriptum eſt triangulo D E F. quia anguli A. B. C. tangunt latera D E. E F. D F.

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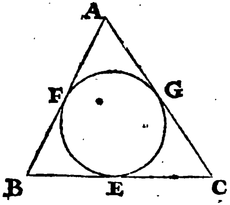
2. *Similiter & figura circum figuram deſcribi dicitur, cum ſingula ejus quæ circumſcribitur, latera, ſingulos ejus figuræ angulos tetigerint; circum quam illa deſcribitur.*

Ut triangulum D E F. dicitur propriè deſcribi circa triangulum A B C. quia ſingula latera majoris trianguli, ſingulos angulos minoris tangunt. Dixi propriè, quia ut impropriè dicatur figura aliqua inſcribi vel deſcribi, ſufficit, ut bene advertit illuſtriſſimus Princeps Fluſſates Candalla ut nullus ſit angulus interioris figuræ, qui non tangat angulum aliquem, vel latus vel planum figuræ exterioris; & eo ſenſu intelligendæ ſunt propoſitiones Hypſiclis lib. 15. elementorum.

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3. *Figura autem rectilinea, in circulo inſcribi dicitur, cum ſinguli, ejus figuræ, quæ inſcribitur, anguli, tetigerint circuli peripheriam.*



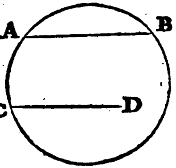
4. *Figura vero rectilinea circa circulum deſcribi dicitur, cum ſingula latera ejus quæ circumſcribitur, circuli peripheriam tangunt.*

5. *Similiter & circulus in figura inſcribi dicitur, cum circuli peripheria ſingula latera tangit ejus figuræ in qua inſcribitur.*

6. Cir-

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6. Circulus autem circum ſiguram deſcribi dicitur, cum circuli peripheria, ſingulos tangit ejus figuræ, quam circumſcribit, angulos.

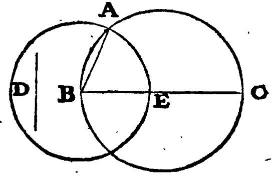


7. Recta in circulo accommodari, ſeu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint.

Sic A B. dicitur in circulo accommodata non vero C D.

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PROPOSITIO I.



Prob. 1. *In dato circulo* A B C. *accommodare rectam* B A. *æqualem datæ rectæ* D. *quæ circuli diametro* B C. *non ſit major.*[a]

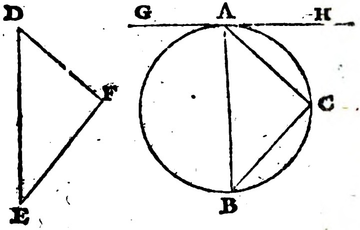
a 7. Def. 4.

Dati circuli ducas diametrum B C. ſi data recta D. æqualis ſit diametro B C. factum eſt quod petitur. Si D. minor ſit
b 3. 1. diametro: [b] abſcindatur B E. æqualis ipſi D. & centro B. ſpatio B E. fiat circulus E A. dicta
c 7. Def. jam recta B A. coaptata erit [c] in
d 15. Def. 1. circulo BAC. & [d] æqualis ipſi BE. & conſequenter ipſi D. Q. E. F.

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PRO-

PROPOSITIO II.



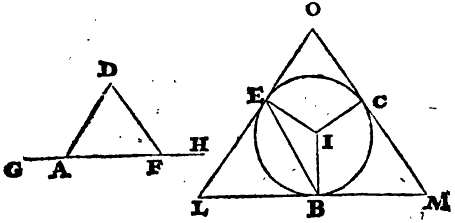
In dato circulo A B C. *triangu-* Prob. 2.
lum A B C. *deſcribere, dato triangulo* D E F. *æqui angulum.*

[a] Fiat tangens GH. ad punctum a 16. 3.
A. fiat angulus H A C. [b] æ- b 23. 1.
qualis angulo E. & G A B. angulo F. ducta recta B C. factum erit quod petitur.

Prob. Angulus H A C. æqua-
lis eſt [c] angulo B. & ſimiliter an- c 32. 3.
gulus G A B. angulo C. ergo & angulus E. angulo B. & angulus F. angulo C. & conſequenter angulus
D. angulo A. [d] æqualis. Ergo trian- d Cor. 2. 32. 1.
gulum triangulo æqui angulum in dato circulo inſcriptum. Q. E. F.

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PROPOSITIO III.



Prob. 3. *Circa datum circulum* BCE. *deſcribere triangulum* LMO. *æquiangulum dato triangulo* D. F. A.

a 23. 1. Dati tranguli latus AF. produc in G. & H. angulo D F H. æqualis fiat ad centrum angulus CIB. & angulo D A G. angulus

b 11. 1. EIB. & ad puncta EBC. b ducas

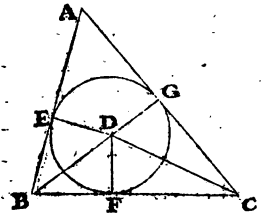
c Ex 16. 3. perpendiculares quæ c tangentes erunt ſcilicet M O. M L. L O. & coëuntes petitum triangulum conſtituent. Quod autem concurrant patet; nam uterque angulorum ad A. & C. eſt rectus: ergo ſi intelligatur duci linea B E. erunt duo anguli verſus L. minores duo-

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duobus rectis: [d] ergo in illam par- d 11. Ax.
tem protractæ tangentes concur-
rent ſimiliterque aliæ in alias par-
tes protractæ: ergo fiet triangu-
gulum circa datum circulum.
Quod autem ſit dato triangulo æ-
quiangulum, ſic probo. In qua-
drilatero C I B M. anguli ad C.
& B. [e] ſunt recti: ergo reliqui e 18. 3.
C I B. CMB. [f] duobus rectis ſunt f 32. 1.
æquales: Sed angulus C I B. æ-
qualis ponitur ipſi D F H. ergo
angulus C M B. æqualis eſt angu-
lo [g] D F A. eodem modo oſtendi g 13. 1.
poteſt in quadrilateris B I E L.
C I E O. angulos L. & O. æqua-
les eſſe angulis A. & D. Ergo
circa datum, &c. Q. E. F.

PROPOSITIO IV.

Prob. 4.



In dato triãgulo AB C. *circulum* G E F. *deſcribere.*

a 9. 1. Divide duos ejus angulos B. & C. bifariam per rectas C D. B D. & ex puncto in quo concurrent

b 12. 1. puta D. b duc perpendiculares DE. DG. D F. ad tria latera dati trianguli. Jam quia triangulorum F C D. G C D. angulus C. unius, ponitur æqualis angulo C. alterius, & uterque angulorum G & F. rectus eſt, & latus C D commu-

c 26. 1. ne: linea D G. c erit æqualis lineæ D F. ſimiliterque oſtendetur rectas D E. DF. eſſe æquales. Poſito ergo centro in D.

d 9. 3. deſcriptus circulus ſpatio DG. d tranſibit per puncta EGF. & quia per coroll. 16. 3. unaquæque linearum AB. BC. CA. tanget circulum, patet perfectũ eſſe propoſitũ.

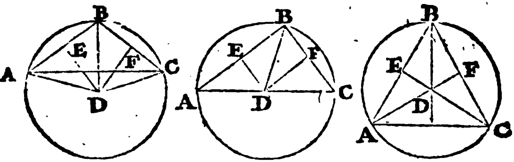
SCHOLIUM.

Hinc cognitis lateribus trianguli, inveniuntur ſegmenta quæ fiunt ad puncta contactus circuli inſcripti. ſcil: ſit AB. 12. BC. 16. AC. 18. *erit* AB. BC. 28. *ſubtrahatur* AC. 18. *æquale* AE. *&* FC. *remanebit* 10. *pro* B E. *&* B F. *adeoque* BE. *vel* B F. *erit* 5. *& per conſequens* F C. *vel* GC. 11. GA. *vel* AE. 7.

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PRO-

PROPOSITIO V.



Circa datum triangulum ABC. *circulum deſcribere.* *Prob.* 5.

Cujuſcunque dati trianguli, duo aliqua latera puta A B. B C. a divide bifariam in E. & F. b ad quæ puncta excitabis perpendiculares quæ coïbunt in D. vel intra triangulum, vel in tertio latere, vel extra (ducta enim EF. fient anguli D E F. D F E. minores duobus rectis: ergo coïbunt) duc præterea rectas D B. D A. D C. Quia triangulorum B E D. A E D. latera BE. EA. ſunt æqualia & D E. commune & anguli ad E. recti, erunt & baſes A D. D B. æquales. Eodem modo c erunt æquales baſes D B. D C. Centro igitur D. ſpatio B D. ductus circulus A B C. tranſibit per puncta A B. C. Circa datum ergo triangulum, circulum deſcripſimus. Q. E. F.

a 10. 1.
b 11. 1.
c 4. 1.

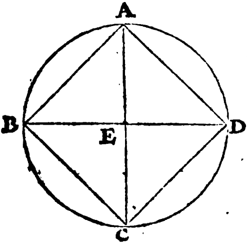
SCHOLIUM.

Hinc etiam patet methodus deſcribendi circulum, qui tranſibit per tria data puncta non in rectum conſtituta.

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PROPOSITIO VI.

Prob. 6.



In dato circulo ABCD. *quadratum deſcribere.*

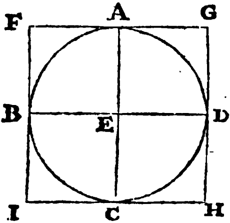
Ducantur duæ diametri A C. B D. ſecantes ſe ad angulos rectos in centro E. & jungantur rectæ BA. BC. CD. DA. & factum eſt quod petitur.

Prob. Quatuor anguli ad centrum E. ponuntur recti & quatuor lineę EA. EB. EC. ED. ęquales. a 4. 1. [a] ergo & quatuor baſes AB. BC. CD. DA. ſunt æquales. Omnia ergo quadrati latera ſunt æqualia. Anguli vero his lateribus contenti ſunt omnes in ſemicirculo: [b] adeoque recti: b 31. 3. Erit igitur ABCD. quadratum circulo inſcriptum. Q. E. F.

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PROPOSITIO VII.



Circa datum circulum, quadratum deſcribere. Prob. 7.

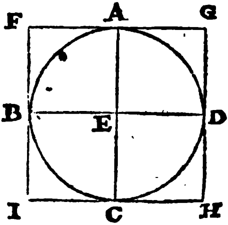
Ductis duabus diametris A C. B D. ſecantibus ſe ad rectos in centro E. per earum extrema ſi ducantur perpendiculares F G. F I. I H. H G. coëuntes petitum dabunt quadratum,

Prob. Anguli quatuor ad E. ponuntur recti, ſicut & anguli ad ABCD. a ergo rectæ FG. BD. HI. ſunt parallelæ, ſimiliterque rectæ FI. AC. GH. b ergo figura F G I H. eſt parallelogramma. Angulus A C H. eſt rectus: c ergo Angulus H G A. eſt rectus, eodem modo oſtendetur angulos F. I. H. eſſe rectos. a 28. 1. b 34. 1. c 34. 1.

De lateribus ſic dico, latus I H. eſt æquale lateri BD. & latus HG. lateri AC. hoc eſt BD. ergo latera I H. H G. ſunt æqualia: ergo quatuor latera ſunt æqualia. Ergo eſt quadratum cujus latera circulum tangunt per coroll. 16. lib. 3. Ergo circa datum, &c. Q. E. F.

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PROPOSITIO VIII.



Prob. 8. *In dato quadrato, circulum deſcribere.*

a 10. 1. Latera quadrati a divide bifariam in ABCD: duc rectas AC. BD. ſecantes ſe in puncto E. quod dico eſſe centrum circuli ſpatio E B. deſcribendi.

Prob. Rectæ A F. I C. ſunt parallelæ

b 33. 1. & equales: ergo rectæ A C. F I. b ſunt parallelæ & æquales, & ſimiliter rectæ A C. H G. eodemque modo rectæ FG.

c 34. 1. I H. c ſunt igitur parallelogramma F E. EI. E H. E G. quare cum æquales. Rectæ B F. F A. A G. ſunt æquales, ipſis

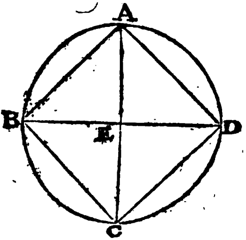
d 14. 1. d B E. E A. E D. rectæ B E. E A. E D.

e 9. 3. erunt & æquales. e Ergo E. eſt centrum, ex quo ſi ſpatio E A. deſcribatur circulus, tanget puncta A B C D. & conſequenter omnia quadrati latera per co-

f 29. 1. roll. pr. 16. l. 3. f In dato ergo, &c. Q. E. F.

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PROPOSITIO IX.



Circa datum quadratum, circulum deſcribere. Prob. 9.

Ducantur diametri A C. B D. ſecantes ſe in puncto E. quod dico eſſe centrum deſcribendi circuli.

Prob. Rectæ A B. A D. ſunt æquales: a ergo & anguli A B D. A D B. Angulus B A D. b eſt rectus, c ergo anguli A B D. A D B. ſunt ſinguli ſemirecti; eodem modo partes angulorum ad A. B. C. D. erunt ſemirecti: ergo omnes inter ſe æquales. d Ergo latera E A. E B. E C. E D. æqualibus angulis ſubtenſa ſunt æqualia. e Ergo E. eſt centrum circuli, qui ſi deſcribatur ſpatio E A. tranſibit per puncta quadrati A B C D. Ergo circa datum, &c. Q.E.F.

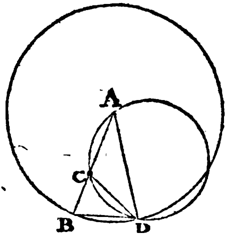
a 5. 1.
b 32. 3.
c 32. 1.
d 6. 1.
e 9. 3.

PRO-

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PROPOSITIO X.

Pr. 30.



Iſoſceles triangulum A B D. *conſtituere, quod habeat utrumque eorum qui ad baſim ſunt, angulorum* B. & D. *duplum reliqui* A.

a 11. 1. Sume rectam quamlibet A B. quæ ſic a dividatur in C. ut rectangulum ſub A B. B C. æquale ſit quadrato rectæ A C. tum centro A. ſpatio B.
b 1. 4. ducatur circulus; in quo b accommodetur recta B D. æqualis ipſi A C. jungaturque recta A D. dico triangulum A B D. fore quæſitum, quod ſic probo.

c 5. 4. Ducta recta C D. c deſcribe circulum A C D. circa triangulum D A C. cum itaque rectangulum ſub A B. B C. æquale ponitur quadrato C A. erit etiam æquale quadrato B D. cum B D. æqualis ponitur ipſi A C. Ergo cum à puncto B. ducatur ſecans BA. recta B D. ab eodem puncto ducta incidens in circulum A C D.

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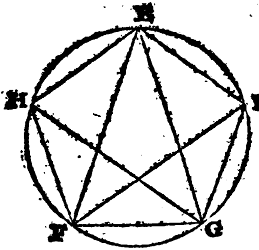
ACD. quorum rectangulum & quadratum ſunt æqualia, BD. tanget d circulum in D. ergo angulus CDB. e æ- d 37. 3.
qualis eſt ipſi A. in alterno ſegmento, e 32. 3.
ergo communi CDA. addito, duo anguli A. & CDA. æquales ſunt duobus BDC. & CDA. hoc eſt toti ADB. vel ABD. Sed angulus externus BCD. duobus internis A. & f 32. 1.
ADC. f æqualis eſt: ergo idem BCD. erit æqualis ipſi CBD. vel ADB. ergo g 6. 1.
rectæ DC. DB. g æquales, cum æquales angulos ſubtendant. Sed BD. ponitur æqualis ipſi CA. ergo CD. CA. æquales erunt: ergo anguli A. & h 5. 1.
CDA. h æquales. Ergo externus angulus BCD. duplus eſt ipſius A. ergo ejuſdem quoque dupli ſunt CBD. ADB. cum ſinguli externo BCD. æquales ſint. Triangulum ergo, &c. Q. E. F.

Corollarium.

Cum tres anguli A. B. D. ſimul conſtituant $\frac{5}{5}$ duorum rect. hoc eſt duos rectos, liquet A. eſſe $\frac{1}{5}$ duor. rectorum.

PRO-

PROPOSITIO XI.



Pr. 11. *In dato circulo* EHFGI. *pentagonum æquilaterum & æquiangulum inſcribere.*

a 10. 4. [a] Fiat triangulum Iſoſceles quicunque, cujus anguli ad baſim ſint dupli ejus qui ad verticem & ipſi æqui angulus [b] inſcribatur in dato circulo E F G. (b 2. 4.) Angulos ad baſim divide biſariam rectis I F. H G. jam quinque puncta E. H. F. G. I. junge lineis totidem, & factum eſſe quod petitur, ſic probo. Quinque anguli E F G. F G H. HGF.

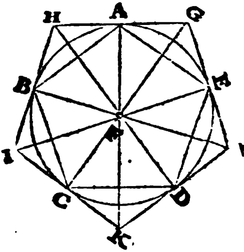
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HGF. IFG. EFI. ponuntur æquales: [c] ergo arcus quibus inſiſtunt, ſunt æquales [d] Ergo æquales rectæ quæ æquales peripherias ſubtendunt. Arcus EH. æqualis eſt arcui FG. ergo ſi addas communem BF. erunt peripheriæ EHF. HFG. æquales: ergo & reliqua ſegmenta FG IE. GI. EH. æqualia: [e] ergo anguli EHF. PFF. æquales. Idemque dicendum de reliquis. Ergo pentagonum æquilaterum & æquiangulum inſcriptum. Q. E. F.

c 26. 3.
d 29. 3.
e 27. 3.

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PROPOSITIO XII.



Pr. 12. *Circa datum circulum* ABCD. *pentagonum* GHIKL. *æquilaterum & æquiangulum deſcribere.*

Quaſi juxta propoſitionem 11. inſcripſiſſem pentagonum in dato circulo, reperiam centrum F. & notabo in peripheria quinque linearum FA. FB. &c. quinque puncta angularia ABCDE.
a corol. 16. 3. & ab iiſdem punctis a ducam tangentes quæ
b 11. Ax. b concurrent in punctis GHIKL. à quibus ſi duxero ad centrum rectas GF. IF. ſic demonſtrabo factum eſſe quod petitur. Et primo quidem quod anguli omnes ſint æquales. In
c 32. 1. quadrilatro AFBH. quatuor anguli c valent quatuor rectos cum cujuſlibet trianguli AHF. HFB. tres anguli valeant duos rectos: ſimiliterque in quadrilatero BFCI. & ſic de aliis: ergo cum anguli A. & B. ſint recti, anguli AHB. AFB. valent duos rectos, ſimiliterque anguli BIC. CFB. & ſic de aliis. Sed anguli
d 27. 3. AFB. BFC. ſunt d æquales ob æquales arcus, ergo

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ergo reliqui H. & I. ſunt æquales, idemque dicendum de aliis. Ergo omnes pentagoni anguli ſunt æquales.

Quod autem latera etiam ſint æqualia ſic probo. Quadratum FI. e eſt æquale quadratis tam ipſarum FB. BI. quam ipſarum IC. CF. ſublatis ergo quadratis æqualium FB. FC. remanent æqualia quadrata BI. IC. ergo rectæ BI. IC. ſunt æquales. Nunc anguli FBI. FCI. & continentia latera ſunt æqualia: ergo f anguli BIF. FIC. ſunt æquales. Eodemque modo dicam de triangulis CFK. KFD. & de aliis omnibus. Ergo cum anguli BFD. CFD. g ſint æquales, & anguli IFC. CFK. ſint eorum dimidia, æquales erunt anguli IFC. CFK. Ergo cum in triangulis IFC. CFK. anguli IFC. FCI. æquales ſint duobus angulis CFK. FCK. alter alteri & latus FC. commune, reliqua latera h erunt æqualia. Ergo rectæ IC. CK. ſunt æquales, & dimidiæ ipſius IK. eodem modo oſtendam IB. eſſe dimidiam ipſius IH. & ſic de aliis: ergo, cum dimidiæ IC. IB. oſtenſæ ſint æquales, erunt tota latera HI. IK. æqualia, idemque dicendum de aliis. Q. E F.

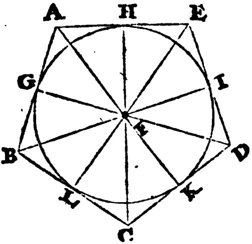
e 47. 1.
f 4. 1.
g 27. 1.
h 26. 1.

Corollarium.

Hinc, ſi in circulo qualiſcunque figura æquilatera & æquiangula fuerit inſcripta, lineæ perpendiculares ad extremitates ſemidiametrorum excitatæ conſtituent figuram totidem laterum & æqualium angulorum circulo circumſcriptam.

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PROPOSITIO XIII.



Pr. 12. *In dato pentagono quod eſt æquilaterum & æquiangulum, circulum inſcribere.*

a 9. 1. [a] Dividantur bifariam duo anguli proximi BAE. ABC. b 11. Ax. rectis A F. B F. quæ [b] coibunt, puta in F. cum nullius anguli medietas valeat rectum. Idem fiat reliquis angulis. Quoniam igitur triangulorum ABF. FBC. æqualia ſunt latera B A. B C. & B F. c Ex. conſt. commune, & anguli ad B. [c] ſunt equales, anguli BAF. BCF. & baſes A F. C F. d 4. 1. [d] erunt æquales. Sed angulus BAF. eſt dimidium angu-

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anguli BAE. ergo quoque BCF. erit dimidium anguli BCD. Eodem modo reliqui anguli bifariam erunt ſecti. Ducantur ſimiliter ex F. ad ſingula pentagoni latera perpendiculares FG. FH. &c. Quia triangulorum GFB. BFL. duo anguli FGB. GBF. duobus FLB. FBL. ſunt æquales, & latus FB. commune, æqualia etiam [e] erunt latera FG. FL. & his FK. FI. FH. quare centro F. ſpatio FG. [f] ſi ducatur circulus, tranſibit per puncta H.I. K.L. exiſtentia in lateribus pentagoni, [g] quę etiam tangent circulũ, cum ſint ſuper extremitate diametri ad rectos conſtitutæ. Q. E. F.

e 26. 1.

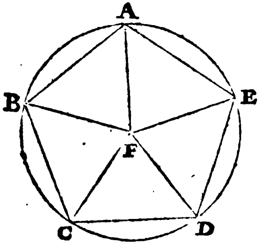
f 15. Def. 1.

g Coroll. 16. 3.

SCHOLIUM.

Hinc duo ſequuntur. 1. omnes angulos cujuſcunque figuræ æquilateræ & æquiangulæ bifariam ſecari per lineas à puncto ductas in quo coeunt duæ rectæ proximos angulos biſecantes. 2. eadem methodo in quacunque figura æquilatera & æquiangula circulum deſcribere.

PROPOSITIO XIV.



Pr. 14. *Circa datum pentagonum quod eſt æquilaterum & æquiangulum, circulum deſcribere.*

a 9. 1. Angulos A. & E. [a] divido bifariam rectis AF. FE. quæ alicubi [b] concurrent, puta in F. hinc ad reliquos angulos duco rectas FD. FC. FB. quas eos ſecare bifariam probatur ut in proxima propoſitione per prop. 26. 1. Ergo cum anguli totales ponantur æquales, æquales erunt dimidii, & [c] conſequenter æqua-les

b 11. Ax.

c 1.

les F A. F B. hiſque æquales omnes rectæ F C. F D. F E. Ergo centro F. ſpatio F A. deſcriptus circulus tranſibit per angulos pentagoni, nec ullum ejus latus [d] ſecabit, cum omnia cadant intra circulum. Q. E. F. d 2. 3.

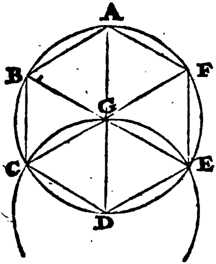
SCHOLIUM.

Eodem prorſus modo circa quamlibet figuram æquilateram & æquiangulam circulus deſcribetur.

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PROPOSITIO XV.

Pr. 15.



In dato circulo, hexagonum, & æquilaterum & æquiangulum inſcribere.

Sit diameter A D. centro D. ſpatio ſemidiametri D G. fiat circulus C G E. ſecans datum circulum in C. & E. per centrum G. ductis C F. E B. jungantur A B. B C. C D. &c. eritque inſcriptum hexagonum æquilaterum & æquiangulum.

Prob. Rectæ G C. G D. à centro G. & rectæ C D. D E. à centro D. ſunt æquales, ergo triangulum D G C. eſt æquilaterum, a Ergo & æquiangulum.

a 51.

Hi

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Hi tres anguli, b valent duos b 32. 1.
rectos: ergo quilibet eorum eſt
pars tertia duorum rectorum.
Similiterque angulus D G E.
Ergo cum C G E. E G F. c va- c 13. 1.
leant duos rectos. E G F. erit
etiam pars tertia duorum recto-
rum. Sed illis d æquales ſunt an- d 15. 1.
guli ad verticem. Ergo ſex an-
guli ad centrum G. ſunt æquales.
Ergo omnes rectæ & circumfe-
rentiæ A B. B C. &c. quibus in-
ſiſtunt e ſunt æquales. Eſt ergo e 26. &
hexagonum æquilaterum. Quod 29. 3.
vero ſit æquiangulum patet, cum
omnium angulorum medietates
ſint oſtenſæ æquales & conſtare
duabus tertiis duorum rectorum.

Coroll. Hexagoni latus, æquale eſt ſemidiametro.

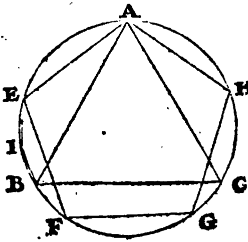
SCHOLIUM.

Hinc facillime triangulum æquilaterum in circulo deſcribetur ductis rectis A C. A E. C E.

PRO-

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PROPOSITIO XVI.



Pr. 16. *In dato circulo quindecagonum & æquilatorum & æquiangulum, deſcribere.*

a 11. 4. [a] Inſcribe in dato circulo pentagonum æquilaterum AEFGH. & eidem ad pun-
b 2. 4. ctum A. [b] inſcribe triangulum æquilaterum ABC. hoc poſito cum tertiam partem citcumfe-
c 26. & 28. 3. rentiæ [c] ſubtendat AB. hoc eſt quinque quindenas, duo vero pentagoni latera, AE. EF. earundem quindecimarum ſubtendant

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dant ſex. Si ab ipſis A E. E F. ſubtentibus ſex, ipſam A B. ſubtendentem quinque tollas, ſupererit B F. ſubtendens unam decimamquintam totius. Ergo ſi quatuordecim ei æquales in circulo [d] accommodentur, erit [d] 1. 4. quindecagonum æquilaterum & æquiangulum [e] cum ſinguli an- [e] 27. 3. guli ſubtendant arcus æquales tredecim laterum quindecagoni. Q. E. F.

SCHOLIUM.

Omnes propoſitiones hujus libri cum ſunt problemata ejuſdem valoris cenſeri poſſunt, quamvis à quibuſdam inter præcipuas numerantur. 5. & 15.

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EVCLIDIS
ELEMENTUM V.

Hujus Elementi quinti Vitruvius autorem prædicat Eudoxium Gnidium, qui Platonem comitatus eſt in Ægyptum.

DEFINITIONES.

Pars eſt magnitudo magnitudinis, minor majoris, cum metitur majorem.

Id eſt, quæ aliquoties ſumpta, majorem ipſam præciſè conſtituit: ſic unitas, eſt pars ternarii, quia ter ſumpta facit ternarium. Atque hæc eſt pars propriè dicta & quæ vocatur *Aliquota*. Impropriè verò dicta pars, eſt *quæ aliquoties ſumpta, vel ſuum totum excedit, vel ab eo deficit:* ſic

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ſic binarius numerus, eſt impropriè dicta pars ſeptenarii, quia ter ſumptus, deficit: quater autem ſumptus excedit: atque hæc pars dicitur *Aliquanta.* Imo Euclides libro 7. non vocat partem, ſed partes, & benè quia quatuor non eſt pars numeri ſex, ſed ejus duæ partes tertiæ. In generè ſic poſſet definiri. *Pars eſt minor & homogenea quantitas, quæ aliquoties repetita, metitur vel excedit ſuum totum.*

Similiter & ſi definitio Partis, prout traditur ab Euclide, tantum conveniat quantitati continuæ, quæ ſola propriè ſecundùm Philoſophum appellatur Magnitudo, cùm tamen numeros ſuis quoque conſtitui partibus dubium ſit nemini, ſic forte commodius potuiſſet exprimi. *Pars eſt minor quantitas, quæ metitur majorem.* Ut ut ſit, in ſequentibus, partis nomine utar, cum in quan-

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titate continua, tum in diſcreta; imò brevitatis gratiâ frequentius utar numeris, quorum tamen loco poterit quilibet magnitudines tot palmorum intelligere quot numeris exprimentur.

2. *Multiplex autem eſt major quantitas, quam metitur minor.*

Multiplex nil aliud eſt quam eadem quantitas aliquoties repetita.

3. *Ratio eſt duarum quantitatum ejuſdem generis, mutua quædam ſecundùm menſuram habitudo.*

Quod Euclidis dixit λόγος hoc Campanus vertit *Proportio*, melius alii *Ratio*. Senſus vero hic eſt, quando duæ quantitates ejuſ-

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ejuſdem generis, ut duo numeri, duæ lineæ, duæ ſuperficies, duo ſolida (nec enim linea cum ſuperficie, aut linea alba cum ſonora, ut ſic, poſſent conferri, cum ſint diverſi generis) inter ſe comparantur; ſecundùm capacitatem hoc eſt exceſſum, defectum aut æqualitatem, appellatur hæc comparatio aut habitudo mutua Ratio. Obſervabis verò, requiri ſemper duas quantitates: nihil enim habet rationem ad ſeipſum, & decempeda ſolitariè conſiderata, nec major eſt, minor, aut æqualis.

Hæc porrò omnis comparatio in capacitate quantitatis fundatur, ſecundùm quam una quantitas aliam continet vel accuratè, vel ex parte tantùm, vel cum exceſſu. Cùm autem in omni ratione duo ſint termini *Antecedens* & *Conſequens* qui ad invicem referuntur: Ille in nomi-

nativo efferri ſolet, hic in alio caſu: exempli gratia linea ſex palmorum eſt dupla lineæ trium: antecedens eſt linea ſex palmorum; conſequens, lineæ trium. Exceſſus antecedentis ſupra conſequentem vel conſequentis ſupra antecedentem dicitur *Differentia terminorum*. *Ratio Rationalis* eſt quæ eſt inter quantitates commenſurabiles & numeris poteſt exprimi, ut ratio dupla, tripla, &c. *Ratio Irrationalis* eſt ea quæ eſt inter magnitudines quarum nulla eſt communis menſura quæ ullo numero poſſit exprimi: exempli gratia inter latus quadrati & ejus diametrum.

4. *Proportio eſt rationum ſimilitudo.*

Græcè dicitur ἀναλογία, ſenſus verò hic eſt. Quemadmodum comparatio capacitatis duarum quantitarum dicitur ratio:

tio: Ita ſimilitudo duarum vel plurium rationum dicitur Proportio. Ex gr. Cum ſimilis ſit ratio 12. ad 4. quæ 9. ad 3. ideo dico inter has quantitates eſſe proportionem, quia eſt ſimilitudo rationum.

Proportio divitur in *Arithmeticam*, *Geometricam*, & *Muſicam*. *Arithmetica eſt quando tres vel plures numeri per eandem differentiam progrediuntur*, ut hi numeri 4. 7. 10. eſt enim differentia 4. & 7. æqualis differentiæ 7. & 10. hæc proportio dicitur Arithmetica quia invenitur inter numeros in ordine ſuo naturali ſumptos puta 1. 2. 3. 4. 5. &c.

Geometrica eſt ſimilitudo rationum inter tres, vel plures quantitates ut inter numeros 2. 6. 18. eſt enim ratio 2. ad 6. ſimilis rationi 6. ad 18. nam utraque ratio eſt tripla. Hæcque

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ſola eſt propriè dicta proportio, & quam hic definit Euclides.

Proportio Muſica eſt quando tres magnitudines ita ordinantur ut eadem ſit ratio primæ ad tertiam, quæ differentiæ primæ & ſecundæ, ad differentiam ſecundæ & tertiæ, ut 3. 4. 6. Sunt in proportione muſica, quia eadem eſt ratio primi numeri 3. ad tertium 6. quæ differentiæ primi & ſecundi, quæ eſt 1. ad differentiam ſecundi & tertii, quæ eſt 2. dicitur vero harmonica, quia conſonantes facit ſonos, inter quos invenitur.

5. *Rationem habere inter ſe quantitates dicuntur, quæ poſſunt multiplicatæ ſeſe mutuo ſuperare.*

Quia ratio eſt duarum quantitatum ejuſdem generis mutua ſecundùm menſuram habitudo, proptereà quantitates quæ ratio-

rationem habent inter ſe, debent eſſe tales ut ſe mutuo ſuperare poſſint: nam quantitas quæ metitur alteram, poteſt eam ſuperare hinc.

Colligitur 1. inter lineam & ſuperficiem, inter ſuperficiem & corpus, inter lineam finitam & infinitam, inter angulum rectilineum & contactus, nullam eſſe rationem, quia quantumuis horum unum multiplices, nunquam tamen aliud ſuperabit.

Coll. 2. Inter diagonalem & latus quadrati eſſe rationem, quia ita poteſt multiplicari ut latus excedat diagonalem, ſed hæc ratio dicitur irrationalis quia non poteſt exprimi numeris.

Coll. 3. Inter curuilinea & rectilinea eſſe rationem cum inter ea ſit æqualitas & inæqualitas: nam Hippocrates Chius Lunu-

lam creſcentem, & Archimedes Parabolam quadravit, & Proclus inter angulos rectilineos & curuilineos æqualitatem demonſtravit lib. 3. in primum Euclid. ad 12. axioma.

6. *In eadem ratione quantitates dicuntur eſſe, prima ad ſecundam, & tertia ad quartam, cum primæ & tertiæ æquemultiplicia, à ſecundæ & quartæ æquemultiplicibus, qualiſcunque ſit hæc multiplicatio, utrumque ab utroque vel unà deficiunt, vel unà æqualia ſunt, vel unà excedunt, ſi ea ſumantur, quæ inter ſe reſpondent.*

A Signo oſtendit Euclides quomodo poſſimus cognoſcere utrũ quatuor quantitates ſint

in

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in eadem ratione, 1°. Æquemultiplica, inquit, primam quantitatem & tertiam. 2°. Æquemultiplica ſecundam & quartam. 3°. conferas multiplicem primæ cum multiplici ſecundæ, & multiplicem tertiæ cum multiplici quartæ; & vide, utrum quotieſcunque multiplex primæ deficit à multiplici ſecundæ, vel æqualis eſt, vel excedit, etiam multiplex tertiæ tunc deficiat à multiplici quartæ, vel æqualis ſit vel excedat: tunc enim ſi id fiat, certò concludas, has quatuor quantitates eſſe in eadem ratione, ſi non fiat, nega eſſe.

8	6	12	9
4	2	6	3
A.	B.	C.	D.

Exemplum: volo ſcire utrum hæ quantitates A. B. C. D. ſint in eadem

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eadem ratione: 1°. æquemultiplico A. & C. puta per binarium. 2°. æquemultiplico B. & D. puta per ternarium, ut factum vides ſuperius. 3°. confero multiplicem primæ 8. cum multiplici ſecundæ 6. & multiplicem tertiæ 12. cum multiplici quartæ 9. & video non tantùm multiplicem ſecundæ deficere à multiplici primæ, ſed multiplicem quartæ deficere à multiplici tertiæ.

12	12	18	18
4	2	6	3
A	B	C	D.

Deinde iterum æquemultiplico A. & C. puta per ternarium: ſimiliter æquemultiplico B. & D. puta per ſenarium (eadem eſt ratio de quocunque numero per quem æquemultiplices) tum video multiplicem primæ æqualem eſſe multiplici ſecundæ: & multipli-

tipli-

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tiplicem tertiæ multiplici quartæ.

8	16	12	24
4	2	6	3
A	B	C	D.

Tertio æquemultiplico A. & C. puta per binarium, æquemultiplico etiam B. & D. puta per octonarium & adverto multiplicem primæ 8. deficere à multiplici ſecundæ 16. & multiplicem tertiæ 12. à multiplici quartæ 24. & quia qualitercunque æquemultiplicem illas quantitates, ſemper ſe habet multiplex primæ ad multiplicem ſecundæ, ut ſe habet multiplex tertiæ ad multiplicem quartæ, id eſt ſimul deficiunt vel excedunt vel ſunt æquales, propterea concludo eſſe quatuor illas quantitates proportionales & earum primam in eadem ratione eſſe ad ſecundam in qua eſt tertia ad quartam.

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16	15	24	25
4	3	6	5
A	B	C	D.

Alterum exemplum. Proponantur aliæ quatuor A B C D. 1°. æquemultiplico A. & C. puta per quaternarium. 2°. æquemultiplico B. & D. puta per quinarium. 3°. Video multiplicem primæ 16. ſuperare multiplicem ſecundæ 15. multiplicem verò tertiæ 24. ſuperari à multiplici quartæ 25. quare concludo duas quantitates non eſſe in eadem ratione, quia ſi eſſent in eadem ratione, quadruplum tertiæ ſuperaret quadruplum 4æ. Sicut quadruplum primæ, ſuperat quadruplum ſecundæ. Id enim fieri debet qualiſcunque ſit multiplicatio. Quare licet duplum primæ ſuperet duplum ſecundæ; & ſimiliter duplum tertiæ ſuperet duplum

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quar-

quartæ. Tamen non poteſt inde colligi quod ſint proportionales; quia ut ſint proportionales oportet ita fieri facta quavis multiplicatione.

SCHOLIUM.

Hæc ſunt quæ ad verba & ſenſum Euclidis nunc occurrunt. Quod ad rem ipſam, nunquam judicavi definitionem illam poſſe inſervire tyronibus: cum tradatur per obſcurius. Sic itaque illam aliter enuncio. *Quatuor quantitates dicuntur eſſe proportionales, cum prima eodem modo continet ſecundam, vel continetur à ſecunda, quo tertia continet quartam vel continetur à quarta.* Nam quatuor quantitates eſſe proportionales, eſt primam ita ſe habere ad ſecundam, ſicut tertia ſe habet ad quartam: hoc autem aliud nihil eſt, quam primam ita eſſe majorem vel minorem ſe-

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cunda, ſicut tertia major eſt vel minor quarta. Si autem res ita ſe habet, prima eodem modo continebit ſecundam, vel à ſecunda continebitur, quo tertia continebit quartam vel à quarta continebitur. Igitur quatuor quantitates dicuntur proportionales, cum prima eodem modo continet ſecundam, vel continetur à ſecunda, quo tertia continet quartam vel continetur à quarta.

Nota hanc definitionem convenire tum quantitatibus rationalibus, tum irrationalibus. Supereſt tantum explicandus ille modus continentiæ vel contentionis qui dicitur idem. Ille autem modus dicitur idem dupliciter, primo cum prima quantitas continet ſecundam aut continetur à ſecunda toties exacte, quoties tertia continet quartam, aut continetur à quarta exacte, ita ut nulla pars ſuperſit v. g. linea duorum pedum

toties

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toties continet lineam unius pedis, quoties linea 6. pedum continet lineam 3. pedum. Similiterque linea unius pedis toties continetur in linea duorum pedum, quoties linea 3. pedum continetur in linea 6. pedum. Et proinde 4. illæ lineæ dicuntur proportionales.

Secundo, ille modus continentiæ vel contentionis dicitur idem cum prima ſecundam, & tertia quartam æque continet; & præterea eandem partem, vel eaſdem partes; vel cum prima, cum tali ſui parte aut talibus partibus continetur in ſecunda, quoties tertia cum eadem, aut talibus partibus continetur in quarta. Ut linea 10. pedum continet toties lineam 3. pedum & talem inſuper ejus partem, quoties lineam 6. pedum qualemve ejus partem continet linea 20. pedum. Nam linea 10. continet ter lineam trium pedum

& inſuper trientem ipſius ternarii, ſicut linea 20. pedum continet ter 6. & inſuper trientem ipſius ſenarii. Similiter linea 12. pedum toties continet lineam 5. pedum & tales ejus partes, quoties lineam 10. pedum qualeſve ejus partes continet linea 24. Rurſus linea 3. pedum cum tali ſui parte continetur in linea 10. pedum ſicut linea 6. pedum cum tali ſui parte continetur in linea 20. pedum. Similiter linea 5. pedum cum talibus ſui partibus continetur in linea 12. pedum, ſicut linea 10. pedum cum talibus ſui partibus continetur in linea 24. pedum.

7. *Eandem autem habentem rationem quantitates, vocentur proportionales.*

Nam quæ habent eandem rationem, habent rationum ſimili-

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militudinem ſeu proportionem. Quod ſi proportio non interrumpitur, dicitur continua proportio, qualis eſt in his numeris 4. 8. 16. 32. qui propterea dicuntur continue proportionales: ſecus autem dicuntur tantum proportionales ut 4. 2. 6. 3.

8. *Cum vero æquemultiplicium, multiplex primæ, exceſſerit multiplicem ſecundæ: at multiplex tertiæ, non exceſſerit multiplicem quartæ: tunc prima ad ſecundam, majorem rationem habere dicetur, quam tertia, ad quartam.*

16.	15.	24	25.
4.	3.	6.	5.
A	B	C	D.

SCHOLIUM.

Vel potius ut in ſcholio ad definitionem 6. à contrario

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tunc prima ad ſecundam majorem rationem habet quam tertia ad quartam cum primum antecedens magis continet ſuum conſequens quam alterum antecendens ſuum conſequens, & contra.

9. *Proportio vero in tribus ad minimum terminis conſiſtit.*

Cum proportio ſit rationum ſimilitudo: ratio autem ſit duarum magnitudinum ejuſdem generis comparatio, duarum una dicitur antecedens, alia conſequens: in proportione, ad minimum duo requiruntur antecedentia, & duo conſequentia: quia tamen medius terminus poteſt eſſe conſequens primæ & antecedens ſecundæ rationis, propterea proportio poteſt eſſe in tribus terminis, nimirum quæ continua eſt. ut 16. 8. 4. quę vero non eſt continua.

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tinua, poſtulat quator terminos ut 16. 4. 12. 3.

10. *Cum autem tres quantitates proportionales fuerint: prima ad tertiam dicitur duplicatam habere rationem, eam quam habet ad ſecundam. At cum quatuor quantitates continue proportionales fuerint: prima ad quartam dicitur triplicatam habere rationem, eam quam habet ad ſecundam: & ſemper deinceps uno amplius, quandiu proportio extiterit.*

Differunt ratio dupla & ratio duplicata, itemquem ratio tripla, & ratio triplicata, ut iſta oſtendunt exempla.

64. 16. 4. 1.
A. B. C. D.

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Primum ſint quatuor quantitates A. B. C. D. continue proportionales, nulla ex ipſis erit ratio dupla vel tripla, & erit nihilominus in ipſis una ratio duplicata & una triplicata: quia ratio primæ ad ſecundam erit inter primam & tertiam duplicata. Erit porrò illa ratio primæ ad ſecundam quadrupla. Quartæ ad tertiam quadrupla duplicata, id eſt quater quadrupla ſeu ſexdecupla. Primæ ad quartam quadrupla triplicata, id eſt quater quater quadrupla, id eſt quater ſexdecupla, id eſt, ſexagequadrupla.

Secundum. Sint quantitates quatuor E. F. G. H. continue proportionales, erit prima ſubdupla ſecundæ. Secunda tertiæ. Tertia quartæ: Erit tamen ratio primæ ad tertiam dupla rationis quam habet prima ad ſecundam.

(above E. F. G. H.: 1. 2. 4. 8.)

Erit

Erit item ratio primæ ad quartam, tripla rationis quam habet prima ad ſecundam, nec tamen erit prima dupla tertiæ, ſed ejus ſubquadrupla: nec prima eſt tripla quartæ, ſub ejus ſuboctupla.

Uno verbo diſcrimen aperio. Inter duas quantitates non dicitur eſſe ratio dupla niſi una præciſe bis alteram contineat: dicitur autem eſſe ratio duplicata, quamcunque habeant inæqualitatem, modo bis ea repetatur comparatio quæ eſt inter primum & ſecundum terminum: & triplicata, ſi tertio eadem inſtituatur.

II. *Homologæ quantitates dicuntur eſſe antecedentes quidem antecedentibus, conſequentes vero conſequentibus.*

1. 4. 8. 32.

Si proportionales ſunt ABCD. & ut prima ad ſecundam, ita tertia

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tertia ad quartam: homologæ dicenter prima & tertia inter ſe, ſecunda item & quarta inter ſe, quia eaſdem vices gerunt prima & tertia, & ſimiliter ſecunda & quarta.

Sequuntur modi argumentandi in proportionibus, qui inferius ſuis locis demonſtrabuntur.

12. *Alterna ratio, eſt ſumptio antecedentis ad antecedentem, & conſequentis ad conſequentem.*

Quia Geometræ quinque diverſas concluſiones colligunt ex una quatuor quantitatum proportione, propterea quinque modos illarum concluſionum nunc definit Euclides. Prima eſt alterna, hoc eſt permutata ratio, ſeu permutando quantitates & comparando ipſas antecedentes inter

inter ſe, & ipſas conſequentes inter ſe.

9. 3. 6. 2.
A. B. C. D.

puta ex eo quod proportionales ſunt A B C D. eſtque ut A. ad B. ita C. ad D. inferam ergo permutando ut A. ad C. ita B. ad D.

13. *Inverſa ratio, eſt ſumptio conſequentis inſtar antecedentis ad antecedentem velut conſequentem.*

Secunda ſpecies ſeu modus argumentandi dicitur inverſa ratio, quando conſequens inſtar antecedentis ſumitur, invertendo ſcilicet terminos proportionis, & ad antecedens velut ad conſequens comparatur. Nam quia eſt ut $\overset{9}{A}$. ad $\overset{3}{B}$. ita $\overset{6}{C}$. ad $\overset{2}{C}$. Ergo

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Ergo invertendo inferam ut $\overset{3}{B}$. ad $\overset{9}{A}$. ita $\overset{2}{D}$. ad $\overset{6}{C}$.

14. *Compoſitio rationis, eſt ſumptio antecedentis cum conſequente, velut unius ad ipſum conſequentem.*

Tertia ſpecies dicitur compoſitio rationis cum antecedens ſimul cum conſequente inſtar unius ſumitur, & ad conſequens comparatur. Sic, Quia eſt ut $\overset{9}{A}$. ad $\overset{3}{B}$. ita $\overset{6}{C}$. ad $\overset{2}{C}$. ergo componendo erit, ut $\overset{12}{AB}$. ad $\overset{3}{B}$. ita $\overset{8}{CD}$. ad $\overset{2}{D}$.

15. *Diviſio rationis eſt ſumptio exceſſus, quo conſequentem ſuperat antece-*

dens,

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dens, ad ipſum conſequentem.

Hoc eſt comparatio differentiæ terminorum cum alterútro ipſorum.

9 3 6 2
Ut quia eſt ut A. ad B. ita C. ad D.
erit dividendo ut 6. ad 3. ita 4. ad 2.
vel ut 6. ad 9. ita 4. ad 6.

16. *Converſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſum conſequens.*

Hoc eſt, comparatio unius termini cum differentia terminorum.

9 3 6 2
ut quia eſt ut A. ad B. ita C. ad D.
Erit convertendo rationem.
ut 9. ad 6. ita 6. ad 4.
vel ut 3. ad 6. ita 2. ad 4.
Unde vides quod converſio eſt diviſionis inverſio.

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17. *Ex æqualitate ratio eſt, ſi plures duabus ſint quantitates, & his aliæ multitudine pares, quæ binæ ſumantur in eadem ratione: cum ut in primis magnitudinibus prima ad ultimam, ſic & in ſecundis magnitudinibus, prima ad ultimam ſe habeat.* Vel.

Sumptio extremorum, per ſubductionem mediorum. Ut ſi ſint plures magitudines.

12		4
A	B	C

Et aliæ totidem.

6		2
D	E	F

binæ & binæ in eadem ratione hoc eſt ut

12

A. ad

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12 6
A. ad B. quidpiam. ita D. ad E. quidpiam, & ut B. ad C. ita. E. ad E. erit ex æquo ut in prioribus
12 4
A. ad ultimam C. ita in poſte-
6 2
rioribus D. ad F. Nullum numerum oportet opponere ipſis B. & E. quia hîc non agitur de ipſo, ſed in ſequentibus. Continet autem æqualitas rationis duos modos argumentandi ex proportione plurium, quam quatuor quantitatum: hos duæ ſequentes definitiones explicant.

18. *Ordinata proportio eſt, cum fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam ut conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.*

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Dicitur ordinata proportio, qua duæ partes proportionis eundem ſervant ſuarum rationum ordinem.

12	6	4
A	B	C
6	3	2
D	E	F

Exemplum, eſto utruſque partis prima ratio eſt dupla, ſecunda ratio eſt ſeſquialtera. Concluditur quod ut eſt $\overset{12}{A}$. ad $\overset{4}{C}$. ita eſt $\overset{6}{D}$. ad $\overset{2}{F}$.

19. *Perturbata autem proportio eſt, cum tribus poſitis magnitudinibus, & aliis quæ ſint his multitudine pares: ut in primis quidem magnitudinibus ſe habet ante-*

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cedens

cedens ad conſequentem: ita in ſecundis magnitudinibus antecedens ad conſequentem: ut autem in primis magnitudinibus conſequens ad aliud quidpiam: ſic in ſecundis magnitudinibus quidpiam ad antecedentem.

Hoc eſt, cum ut in primis, prima ſe habet ad ſecundam, ita in ſecundis ſecunda ad tertiam; & ut in primis ſecunda ad tertiam, ita in ſecundis, prima ſe habet ad ſecundam, dicitur hæc proportio perturbata, quia una proportionis pars non ſervat ordinem rationum alterius partis. Exemplum eſto.

12	6	4
A	B	C

6	4	2
D	E	F

 In

In prima propoſitionis parte, ratio dupla præcedit ſeſquialteram.

In ſecunda parte ſequitur,

Concluditur tamen perinde atque in proportione ordinata. Quod ut eſt

	12		4
	A	ad	C
Sic eſt	6		2
	D	ad	F

Axioma ex Tacqueto.

Datis tribus quantitatibus dabilis eſt quarta ad quam tertia talem rationem habet, qualem prima ad ſecundã, hoc eſt, quoties prima continet vel continetur à ſecunda, toties tertia continet vel continetur à quarta.

NOTA.

Cum perplurimæ hujus libri propoſitiones tamquam axiomata haberi poſſunt, ſubinde ſimpliciter nulla adhibitâ demonſtratione declarabo.

Acutiſſimi Tacqueti Methodus laudanda, ſed ne in totum videar diſcedere à Fournier ordinem propoſitionum proſequar.

PRO-

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PROPOSITIO I.

3. 1. 3. 1. *Si ſint quotcunque* Th. 1.
A. E. C. F. *magnitudines quotcun-*
6. 2. *que magnitudinum æ-*
G. H. *qualium numero, ſin-*
gula ſingularum æquemultiplices; quam multiplex eſt unius una magnitudo, tam multiplices erunt & omnes omnium.

Id eſt quia [a] æquemultiplices ſunt A. ad E. & C. ad F. Si A. & C. jungantur in G. ſimiliterque E. & F. in H, quam multiplex erit A. ipſius E. & C. ipſius F. tam multiplex erit G. ipſius H.

a Def. 2. 5.

Prob. Quia tam G. quam H. æquali numero partium continentium ac contentarum augentur.

PROPOSITIO II.

Th. 2. 6 3 4 2 *Si prima* A. *ſecunda* A. B. C. D. B. *æquè fuerit multi-* 9 6 15 10 *plex, atque tertia* C. E. F. G. H. *quartæ* D. *fuerit autem & quinta* E. *ſecundæ* B. *æquè multiplex, atque ſexta* F. *quartæ* D. *erit & compoſita prima cum quinta* E. nempe G. *ſecundæ* B. *æquemultiplex, atque tertia* C. *cum ſexta* F. nempe H. *quartæ* D.

Prob. ex hypotheſi ſecunda B. & quarta D. pari numero continentur in ſuis multiplicibus A. & C. nempe bis. Similiterque eadem ſecunda B. & quarta D. pari numero continentur in ſuis aliis multiplicibus E. & F. nempe ter. Ergo per præcedentem, continebuntur etiam pari numero in multiplicibus collectis, hoc eſt ſi componantur A. & E. ut fiat G. ſimiliterque F. & G. ut fiat H. quemadmodum G. 15. continet B. 3. quinquies. Ita H. 10. continebit D. 2. quinquies.

PRO-

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RROPOSITIO III.

4 2 6 3 *Si ſit prima* A. *ſecun-* Th. 3.
A B C D *da* B. *æquè multiplex*,
8 12 *atque tertia* C. *quarta*
E F D. *ſumantur autem æquemultiplices* E. *&* F. *primæ* A. *& tertiæ* C: *erit ex æquo ſumptarum, utaque utriuſque æquemultiplex, altera quidem* E. *ſecundæ*, B. *altera autem* F. *quartæ* D.

Prob. Ponuntur B. & D. æqualiter contineri in ſingulis A. & C. ergo æqualiter [a] contineantur etiam in iiſdem pari numero multiplicatis in E. & F. a 1. 5.

PRO-

PROPOSITIO IV.

4 2 6 3 *Si prima* A. *ad ſecun-*
A B C D *dam* B. *eandem habue-*
8 6 12 9 *rit rationem ac tertia ad*
Th. 4. E F G H *quartam: etiam æqui-*
multiplices primæ E. *& tertiæ* G. *ad æquemultiplices ſecundæ* F. *& quartæ* H. *juxta quamvis multiplicationem eandem habebunt rationem, ſi prout inter ſe reſpondent, ſumptæ fuerint.*

Poſita & explicata ſuperius à nobis definitione 6. hanc poproſitionem ſic breviter perſtringo.

Ratio patet præſertim ex ſcholio 6. def. utique idem eſt quatuor quantitates in eadem eſſe ratione & earum æquimultiplicia vel una deficere vel una excedere vel una ęqualia eſſe, quemadmodum idem eſt & vel conferre ſingulas B. & D. ad

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D. ad ſingulas A. & C. atque B. & D. æqualiter multiplicatas ad A.& C. pari inter ſe numero multiplicatas.

Corollarium.

Hinc etiam patet veritas rationis converſæ. Nam ſi A. eſt ita majus ipſo B. ſicut C. ipſo D. eſt evidens B. ita minus fore ipſo A. ſicut D. ipſo C. minus eſt. Nec minus foret evidens ſi A. & C. ſumpta eſſent æqualia, aut minora ipſis B. & D.

PRO-

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PROPOSITIO V.

Th. 4. E 4 F 2 *Si magnitudo* A.
C 8 D 4 *magnitudinis* B. *ita*
A 12 B 6 *multiplex fuerit : ut ablata* C. *ablatæ* D. *etiam reliqua* E. *reliquæ* F. *ita multiplex erit, ut tota* A. *totius* B.

Patet. Sit enim A. duplum ipſius B. & pars ablata C. dupla ſimiliter partis ablatæ D. ergo ſi reſidua E. non eſt duplex reſiduæ F. omnes partes totius B. non continentur in omnibus partibus toties A. ſicut totum in toto. Eſt ergo reſidua reſiduæ ita multiplex, ut tota totius

PRO-

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PROPOSITIO VI.

G 2 H 3	G 8 H 12
E 10 F 15	E 4 F 6
A 12 B 18	A 12 B 18
C 2 D 3	C 2 D 3

Si duæ magnitudines A. & B. duarum magnitudinum C. & D. ſint æquemultiplices: & detracta quædam EF. *ſint earundem* CD. *æquemultiplices. Reliquæ* GH. *iiſdem* CD. *aut æquales ſunt aut æquemultiplices.* Tb. 6.

Prob. C. & D. in totis A. & B. & in eorum aliquibus partibus aſſumptis E. & F. æqualiter continentur ex hypotheſi: [a] ergo æqualiter etiam continebuntur in reliquis G. & H. Ergo reliquæ ejuſdem, aut æquales ſunt aut æquemultiplices.

a 5. 5.

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PROPOSITIO VII.

Th. 7. A A *Æquales* A. *&* A.
12 B 12 *ad eandem* C. *eandem habent rationem : & eadem* C. *ad æquales* A. *&* A.
4

Puriſſimum eſt axioma & perclare patet ex ſcholio def. 3. collato cum axiom. 1. lib. 1. Scil. $\frac{A}{B.}$ ratio eſt æqualis eſt rationi $\frac{A}{B.}$

PRO-

PROPOSITIO VIII.

16 8 5 *Inæqualium magnitudi-* *Th.* 8.
A B C *nem A. B. major A. ad*
6 4 8 *eandem C. majorem rationem habet, quam minor B: Et eadem C. ad minorem B. majorem habet rationem, quam ad majorem A.*

Prob. 1ª pars. Si A. eſſet æqualis B: vel ſi A. & B. æqualiter continerent C. eandem rationem haberent a ad C. & C. eandem ad A. & B. per præcedentem: ſed major ponitur A. hoc eſt pluries continere G. ergo per definitionem 8. A. majorem habet rationem ad C. Prob. 2. Et quia C. pluries continetur ab A. quam à B. minorem habebit ad A. rationem quam ad B. per 8. def.

a 6. *Def.* 5.

PROPOSITIO IX.

Th. 9. A B C *Quæ* A.*&* B.*ad eandem*
15 15 4 C. *eandem habent rationem, æquales ſunt inter ſe, & ad quas* AB. *eadem* C. *eandem habet rationem, hæ quoque* AB. *æquales ſunt inter ſe.*

Si enim dicas A. eſſe majus quam B. [a] ergo major erit ratio majoris A. ad eandem C. quàm minoris B. ad eandem C. Item major ratio ipſius C. ad B. quam ad A. quod eſt contra hypotheſim.

a 8. 5.

PRO-

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PROPOSITIO X.

16 8 4 *Earum magnitudinum* Th. 10.
A B C AB. *quæ ad eandem* C. *habent rationem: quæ* A. *rationem majorem habet, hæc major eſt: ad quam autem* B, *eadem* C, *majorem rationem habet, hæc* B. *minor eſt.*

Si enim B. eſſet æqualis aut major quam A. [a] haberent A. a 7. 5.
& B. eandem rationem ad C. vel
B. [b] haberet majorem, quod eſt b 1. 5.
contra hypotheſim. Item ſi C. habet majorem rationem ad A. quam ad B. minor eſt A. quam B. vel utrumque, quod dixi, ſequatur abſurdum. Hæc convertit 8.

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PROPOSITIO XI.

Th. 11. A 9 E 6 C 12 *Quæ eidem*
B 6 F 4 D 8 *ſunt eadem rationes, & inter ſe ſunt eadem.*

Hæc propoſitio nulla videtur ex præmiſſis indigere explicatione: nimirum ſi A B. ratio ſit æqualis rationi E F. eidemque E E. æqualis C D. erit quoque A B. ratio æqualis rationi C D. per 1. axioma lib. 1.

PRO-

PROPOSITIO XII.

4	2	6	3	
A	B	C	D	
	10	5		
	E	F		

Si ſint quotcunque magnitudines proportionales A B C D *quemadmodum ſe habuerit una antecedentium* A. *ad unam conſequentium* B. *ita omnes antecedentes* E. *ad omnes conſequentes* F. Th. 12.

Quod prop. 1. de proportione multiplici demonſtratur, hîc de omni proportione etiam irrationali oſtenditur per eandem primam & defin. 6. ſi ſumantur antecedentium & conſequentium æquemultiplices. Ratio autem generalis eſt, quia A. & B. æquali numero partium tam continentium quam contentarum augentur in E. & F. adeoque quoties A. vel C. continet B. vel D. toties continebit E. ipſum F.

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PROPOSITIO XIII.

Th. 13. A B C D E F *Si prima* A. *ad ſe-*
6 4 3 2 4 3 *cundam* B. *eandem habuerit rationem, quam tertia* C. *ad quartam* D. *tertia verò ad quartam majorem habuerit rationem, quàm quinta* E. *ad ſextam* F. *prima quoque* A. *ad ſecundam* B. *majorem rationem habebit quam quinta* E. *ad ſextam* F.

Res per ſe ex 3. def. clara; utique ratio $\frac{C}{D}$ hoc eſt $\frac{A}{B}$. ex hypot. major eſt ratione $\frac{E}{F}$.
Q. E. D.

PRO-

PROPOSITIO VIX.

2 3 8 12 *Si prima* A. *ad ſecundam* B. *eandem habuerit rationem, quam tertia* C. *ad quartam* D. *prima verò* A. *quam tertia* C. *major fuerit, erit & ſecunda* B. *major quam quarta* D. *Quod ſi prima* A. *fuerit æqualis tertiæ* C. *erit & ſecunda* B. *æqualis quartæ* D. *Si verò minor, & minor erit.* Th. 14.
9 9 9 9
12 8 6 4
A B C D

Sit A. major quam C. ergo [a] ratio A. ad B. hoc eſt C. ad D. major erit ratione C. ad B. adeoque per 10. hujus B. major erit D. idem concludam ſi A. æqualis fuerit vel minor quam C. Q. E. D.

[a] 8. 5.

PRO-

PROPOSITIO XV.

Th. 15. C 25 D 35 *Partes* A *&* B. *cum* A 5 B 7 *pariter multiplicibus* C *&* D. *in eadem ſunt ratione, ſi prout ſibi mutuo reſpondent, ita ſumantur.*

SIt A. pars ipſius C. & B. ipſius D. continet C. toties A. quoties D. continet ipſam B. Quia ergo ut una antecedentium, A. ad unam conſequentium B. ita ᵃ omnes antecedentes C. ad omnes conſequentes D. Ergo ut C. ad D. ita A. ad B.

a 12. 5.

PRO-

PROPOSITIO XVI.

A.	8	B	10	E	4
C.	4	D	5	F	5

Si quatuor magnitudines ABCD. *proportionales fuerint & viciſſim proportionales erunt.* Th. 16.

HOc eſt, ſi ſit A. ad C. ſicut B. ad D. erit permutando vt A. ad B. ita C. ad D.

Prob. Supponamus enim A. continere C. bis, ſicut continet B. ipſum D. ſi dividamus A. & B. bifariam, erit E. æqualis C. & F. æqualis D. ſed ut E. ad F. ſic dupla A. ad B. per 12. Ergo ut dupla A. ad duplam B. ſic C. æqualis ipſi E. ad D. æqualem ipſi F. Q. E. D.

PRO-

PR

4 2 6 3
A B C I
8 6 12 9
E F G H

multiplices
ad æquem
quartæ H.
cationem e
nem, ſi pro
ſumptæ fueri

Poſita &
nobis d
oproſitione
ſtringo.

Ratio pate
io 6. def. uti
quantitates i
& earum æq
eficere vel
qualia eſſe,
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PROPOSITIO XIX.

D	4	F	2		
A	16	B	8		
C	12	E	6		

Th. 19. *Si quemadmodum totum* A. *ad totum* B. *ita ablatum* D. *ſe habuerit ad ablatum* F. *& reliquum* C. *ad reliquum* E. *ut totum* A. *ad* B. *ſe habebit.*

Prob. A. B. D. F. ponuntur
a 16. 5. proportionales; erit ª ergo
b 17. 5. ut B. ad F. ita A. ad D. Ergo ᵇ erit ut F. ad E. ita D. ad C. Ergo ut F. ad D. ita E. ad C. hoc eſt ut tota A. ad totam B. cum poſita ſit A. ad B. ut D. ad F.

Brevius quia aliter omnes partes eſſent majores omnibus partibus, quam totum toto. Idem fere cum quinta.

PRO-

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PROPOSITIO XX.

12 9 6 *Si ſint tres magnitudines*
A BC ABC. *& alia* DEF. *Tb. 20.*
8 6 4 *ipſis æquales numero, quæ*
D E F *bina & in eadem ratione ſumantur* (hoc eſt ut A. ad B. ita D. ad E. & ut B. ad C. ita E. ad F.) *Ex æquo autem prima* A. *quam tertia* C. *major fuerit, erit & quarta* D. *quàm ſexta* F. *major. Quod ſi prima tertiæ æqualis fuerit, erit & quarta æqualis ſextæ, ſin illa minor, hæc quoque minor erit.*

Prob. Sit major A. quam B. [a] ergo major erit ratio ipſius A. ad B. quam C. ad B. ſed ratio A. ad B. æqualis eſt rationi D. ad E. ergo etiam D. ad E. ratio major eſt quam B. ad C. hoc eſt E. ad F. quare D. major erit F. per 10. 5. Haud ſecus concludam ſi A. ipſi C. æqualis ponatur aut minor. Interpretes idem probant de quotcunque magnitudinibus, non de tribus tantum.

a 8. 5.

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PROPOSITIO XXI.

Th. 21.

18	12	4
A	B	C
27	9	6
D	E	F

Si ſint tres magnitudines A B C. *& ipſis æquales numero* DEF. *quæ bina & in eadem ratione ſumantur, fueritque perturbata earum proportio* (hoc eſt ut A. ad B. ſic E. ad F. & ut B. ad C. ſic D. ad E) *Ex æquo autem prima* A. *quam tertia* C. *major fuerit: erit & quarta* D. *quam ſexta* F. *major. Quod ſi prima tertiæ fuerit æqualis, erit & quarta æqualis ſextæ, ſin illa minor, hæc quoque minor erit.*

Prob. Sit A. major quam C. ergo per 8. A. ad B. majorem rationem habebit quam C. ad B. ſed ratio A. ad B. æqualis eſt rationi E. ad F. ergo etiam ratio E. ad F. major erit ratione B. ad C. hoc eſt D. ad E. adeoque per 10. 5. F. minor erit quam D. Idem oſtendetur ſi A. minor vel æqualis fuerit D.

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PROPOSITIO XXII.

12 9 6 8 6 4 *Si fuerint* *Th. 22.*
A B C D E F *quotcunque*
24 18 12 16 12 8 *magnitudines*
G H I L M N ABC. *& alia ipſis æquales numero* DEF. *quæ bina in eadem ratione ſumantur* (hoc eſt ut A. ad B. ita D. ad E. & ut B. ad C. ita E. ad F.) *& ex æqualitate in eadem ratione erunt.* Hoc eſt erit A. ad C. ſicut D. ad F.

Prob. Sumantur ipſarum ABC. æquemultiplicia GHL. & ipſarum DEF. æquemultiplica, LMN. cum ſimplicia ſint in eadem ratione A. ad B. ut D. ad E. & B. ad C. ut E. ad F. a erunt eorum multiplicia G. ad H. & H. ad I. ut L. ad M. & M. ad N. Ergo ſi quotvis magnitudines GHI. & aliæ totidem LMN. binæ ſumantur in eadem ratione quarum b primæ ultimam in utroque ordine ſimul excedunt, æquantur, vel deficiunt, eadem ſimplices erunt in eadem ratione, hoc eſt A. ad C. c ut D. ad F.

a 15. 5.
b 20. 5.
c 6. Def.

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PROPOSITIO XXIII.

18	12	4
A	B	C
27	9	6
D	E	F

Th. 23. *Si fuerint tres magnitudines* A B C. *aliaque ipſis æquales numero* D E F. *quæ bina in eatione ſumantur, fuerit autem perbata eadem ratio* (hoc eſt ſit A. ad B. ut E. ad F. & ut B. ad C. ita D. ad E.) *etiam ex æqualitate in eadem ratione erunt* (hoc eſt ut A. ad C. ita D. ad F.)

a 21. 5. Prob. [a] Si A. excedit C. æquatur vel deficit; D. excedet F.
b 15. 5. æquabitur, vel deficiet. [b] Idemque fiet in æquemultiplicibus.
c 17. Def. Ergo ex [c] æqualitate in [d] eadem
d 6. Def. ratione eſt A. ad C. ita D. ad F.

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PROPOSITIO XXIV.

4	2	6	
A	B	C	
3	10	15	
D	E	F	
	14	21	
	G	H	

Si prima A. *ad ſecundam* B. *eandem habuerit rationem, quam tertia* C. *ad quartam* D. *habuerit autem & quinta* E. *ad ſecundam* B. *eandem rationem quam ſexta* F. *ad quartam* D. *Etiam* G. *compoſita prima cum quinta: ad ſecundam* B. *eandem habebit rationem, quam* H. *tertia cum ſexta, ad quartam* D. Th. 24.

Prob. Ex hypotheſi B. eſt talis pars ſingularum A. & E. qualis eſt D. ſingularum C. & F. Ergo erit quoque B. talis pars compoſitarum A. & E. in G. qualis eſt ipſarum C. & F. compoſitarum in H. a 18. 5.

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PROPOSITIO XXV.

12	4	9	3.
A	B	C	D.
E	3.	F	1.

Th. 25. *Si quatuor magnitudines* ABCD. *proportionales fuerit: maxima* A. *& minima* D. *reliquis duabus* B C. *maiores erunt.*

Nam ſi ab A. 12. demas C. 9. remanebit E. 3. item ſi à B. 4. auferas D. 3. remanebit F. 1. nunc quoniam eſt A. ad B. ita C. ad D. erit quoque dividendo A. ad B. ita E. 3. F. 1. ſed A. major eſt C. ergo & E. major erit F. ergo A. compoſita ex C. & E. plus D. major erit quam B. compoſita ex C. & F. plus C. Q. E. D.

PRO-

PROPOSITIO XXVI.

8 4 5 3 *Si prima* A. *ad ſe-* *Th.* 26.
A B C D *cundam* B. *habuerit majorem rationem quàm tertia* C. *ad quartam* D. *habebit convertendo, ſecunda* B. *ad primam* A. *minorem, quàm quarta* D. *ad tertiam* C.

Hæc & reliquæ octo propoſitiones, cùm non ſint Euclidis, eas non aliter demonſtrabimus quàm indicando propoſitiones Euclidis in quibus virtute continentur.

Hanc vero propoſitionem 4. & 10. hujus elementi contineri, patet manifeſtè.

PRO-

PROPOSITIO XXVII.

Th. 27. 8 4 5 3 *Si prima* A. *ad ſecun-*
A B C D *dam* B. *habuerit majorem rationem, quam tertia* C. *ad quartam* D. *habebit quoque viciſsim prima* A. *ad tertiam* C. *majorem rationem, quam ſecunda* B. *ad quartam* D.

Continetur prop. 16.

PROPOSITIO XXVIII.

Th. 28. 8 4 5 3 *Si prima* A. *ad ſecun-*
A B C D *dam* B. *habuerit ma-*
E 12 F 8 *jorem rationem, quam tertia* C. *ad quartam* D. *habebit quoque compoſita prima cùm ſecunda* E. *ad ſecundam* B. *majorem rationem, quam compoſita tertia cum quarta* F. *ad quartam* D.

Continetur prop. 18.

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PROPOSITIO XXIX.

8 4 5 3 *Si compoſita* E. *prima* *Th.* 29.
A B C D *cum ſecunda, ad ſecun-*
E 12 F 8 *dam* B. *majorem ha-*
buerit rationem quàm compoſita F. *tertia cum quarta ad quartam* D. *habebit quoque dividendo, prima* A. *ad ſecundam* B. *majorem rationem quàm tertia* C. *ad quartam* D.

Continetur propoſitione 17.

PROPOSITIO XXX.

8 4 5 3 *Si compoſita* E. *prima* *Th.* 30.
A B C D *cum ſecunda, ad ſecun-*
E 12 F 8 *dam* B. *habuerit majo-*
rem rationem, quam compoſita F. *tertia cum quarta, ad quartam* D. *habebit per converſionem rationis, prima cum ſecunda* E. *ad primam* A. *minorem rationem, quam tertia cum quarta* F. *ad tertiam* C.

Continetur prop. 19.

PRO-

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PROPOSITIO XXXI.

16	8	4.	9	5	3.
A	B	C.	D	E	F.

Th. 31. *Si ſint tres magnitudines* A B C. *& aliæ ipſis æquales numero* DEF. *ſitque major ratio prima priorum* A. *ad ſecundam* B. *quam prima poſteriorum* D. *ad ſecundam* E. *Item ſecunda priorum* B. *ad tertiam* C. *major quam ſecunda poſteriorum* E. *ad tertiam* F. *erit quoque ex æqualitate major ratio prima priorum* A. *ad tertiam* C. *quam prima poſteriorum* D. *ad tertiam* F.

Continetur prop. 20. & 22.

PRO-

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PROPOSITIO XXXII.

16 8 5 *Si ſint tres magnitudi-* Tb. 32.
A B C *nes* ABC. *& aliæ ipſis*
9 6 4 *æquales numero* DEF.
D E F *ſitque major ratio primæ*
priorum A. *ad ſecundam* B. *quam*
ſecunda poſteriorum E. *ad tertiam*
F. *Item ſecunda priorum* B. *ad ter-*
tiam C. *quam prima poſteriorum*
D. *ad ſecundam* E. *Erit quoque*
ex æqualitate major ratio primæ
priorum A. *ad tertiam* C. *quam pri-*
mæ poſteriorum D. *ad tertiam* F.

Continetur prop. 21. & 23.

PROPOSITIO.XXXIII.

12 6 *Si fuerit major ratio totius* Tb. 33.
A B A. *ad totum* B. *quam ablati*
4 3 C. *ad ablatum* D. *erit &*
C D *reliqui* E. *ad reliquum* F.
8 3 *major ratio, quam totius* A.
E F *ad totum* B.

Continetur propoſitione 18.

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PROPOSITIO XXXIV.

12 8 4. 6 5 3 *Si ſint quot-*
A B C. D E F *cunque magni-*
Tb. 34. *tudines* ABC. *& aliæ ipſis æquales numero* DEF. *ſitque major ratio primæ priorum* A. *ad primam poſteriorum* D. *quam ſecundæ* B. *ad ſecundam* E. *&* B. *ad eundem* E. *major, quam tertiæ* C. *ad tertiam* F. *& ſic deinceps: habebunt omnes priores ſimul* ABC. *ad omnes poſteriores ſimul* DEF. *majorem rationem quam omnes priores* BC. *relicta prima* A. *ad omnes poſteriores,* EF. *relicta quoque prima* D. *minorem autem, quam prima priorum* A. *ad primam poſteriorum* D. *majorem denique etiam quam ultima priorum* C. *ad ultimam poſteriorum* F.

Hujus nullus uſus & facilis demonſtratio ex præcedentibus.

NOTA.

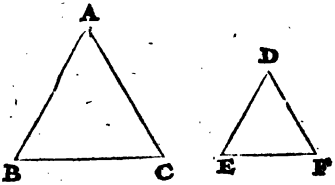
Quidam inter celebriores numerant. 15. 16. 17. 18.

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EVCLIDIS ELEMENTUM VI.

DEFINITIONES.



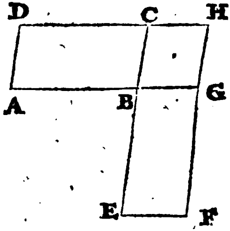
1. *Similes figuræ rectilineæ ſunt, quæ & angulos ſingulos ſingulis æquales habent, atque etiam latera, quæ circum angulos æquales ſunt, proportionalia.*

Duas conditiones requirit, 1. ut anguli ſint æquales ſinguli ſingulis, ut hic A. & D. B. & E. C. & F. 2. ut latera circa æquales angulos ſint proportio-

 nalia,

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nalia, hoc eſt ita ſe habeat B A. ad A C. ut E D. ad D F. quod ſi harum altera deſit, non dicentur ſimiles. Sic quadratum & altera parte longius non ſunt ſimiles figuræ.

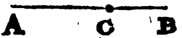


2. *Reciprocæ autem figuræ ſunt, cum in utraque figura, antecedentes & conſequentes rationum termini fuerint.*

Hoc patet maxime in parallelogrammis & triangulis: nam ſi qua ratione AB. eſt ad BG. in

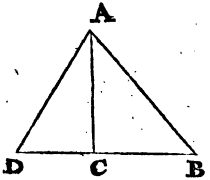
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in eadem ſit BE. ad BC. erunt reciprocæ figuræ, nam in utroque eſt antecedens & conſequens diverſarum rationum.



3. *Secundum extremam & mediam rationem; recta* AB. *ſecta eſſe dicitur, cum ut tota* AB. *ad majus ſegmentum* AC. *ita majus* AC. *ad minus* CB. *ſe habuerit.*

Ob miram ſui utilitatem, hæc proportio, divina communiter appellatur; aſt mirum quod 11. prop. lib. 2. hic inter definitiones annumeratur, niſi velis veritatem jam demonſtratam hic reſumi.



4. *Altitudo cujuſque figuræ, eſt lineæ perpendicularis* A D. *à vertice ad baſim deducta.*

Cum ut ait Ptol. lib. de Anal. menſura cujuſque rei debeat eſſe ſtata, merito Euclides à perpendiculari altitudinem petit cujuſvis figuræ: ſola enim perpendicularis eſt ſtatæ & certæ longitudinis: hanc vero altitudinem lib. 1. vocavit eſſe in iiſdem parallelis.

5. *Ra-*

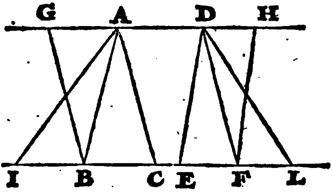
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5. Ratio ex rationibus componi dicitur, cùm rationum quantitates, inter ſe multiplicatæ, aliquam efferint rationem.

Quod Euclides vocat quantitates rationum, ſolent Geometræ vocare Denominatorem. Numerus enim eſt à quo petitur nomen proportionis; ſic 4. eſt denominator rationis quadruplæ: 3. triplæ. Ratio igitur eſt rationibus componi dicitur, quando harum denominatores ſeu quantitates rationum inter ſe multiplicatæ aliquam aliam rationem fecerint. Sic ex ratione dupla & tripla componitur ſextupla, quæ eſt ratio ex rationibus: nam ſex componitur ex denominatore duplæ 3. Inter ſe enim multiplicati faciunt 6. denominatorem rationis ſextuplæ compoſitæ.

PROPOSITIO I.

Th. 1.



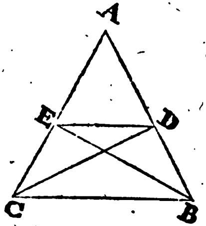
Triangula ABC. DEF. *&* *parallogramma* CG. DF. *quorum*
a Def. 4. [a] *eadem fuerit altitudo* GH. BF. *ita ſe habent inter ſe, ut baſes* BC. EF.

Id eſt, eam inter ſe habent rationem quam baſes. Prob. Triangula ejuſdem altitudinis [a] poſſunt
a Def. 4. inter parallelas conſtitui: [b] tunc
b 36. autem quæ æqualem habebunt baſim, erunt æqualia, quæ majorem majora, quæ minorem minora.
c 15. 5. Idemque [c] eſt de æquemultiplicibus. Ergo abſolutè triangula ſe habent ut baſes; ſimiliterque parallelogramma; cum ſint dupla
d 34. 1. [d] triangulorum.

PRO-

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PROPOSITIO II.



Si ad trianguli ABC. *latus unum* CB. *parallela ducatur* ED. *hæc proportionaliter ſecabit ipſius trianguli latera* AC. AB. *Et ſi trianguli latera, proportionaliter ſecta ſint, recta* D E. *per puncta ſectionis ducta, erit parallela ad reliquum ipſius trianguli latus* C B. Tb. 2.

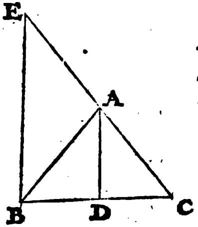
Prob. Ductis duabus rectis EB. DC. a erunt triangula EDC. EDB. ſuper eandem baſim ED. & inter eaſdem parallelas E D. C B. æqualia. b Ergo ut AED. ad ECD. ita A E. ad E C. c (ſunt enim in eadem altitudine) & ut A D E. ad DBE. ita AD. ad DB. d ergo ut AE. ad EC. ita AD. ad DB. 2. Ponantur jam latera AC, AB. proportionaliter ſecta in E. & D. cum AED. ad DEC. eandem habeat rationem, quam ad EDB. (nam eſt ut AE. ad EC. ſic AD. ad DB. cum triangula ſint ejuſdem altitudinis) e erunt DEC. E D B. æqualia, & quia ſunt in eadem baſi f erunt inter parallelas. Q. E. D.

a 37. 1.
b 1. 6.
c Def. 4.
d 7. 5.
e 9. 5.
f 39. 1.

PRO-

PROPOSITIO III.

Tb. 3.



Si trianguli ABC. *angulus* A. *bifariam ſectus ſit: ſecans autem angulum recta* AD. *ſecet & baſim* BC. *baſis ſegmenta* BD. DC. *eandem habebunt rationem, quam reliqua trianguli latera* BA. AC. *& ſi baſis ſegmenta* BD. DC. *eandem habeant rationem, quam reliqua trianguli latera* BA. AC. *recta* AD. *quæ à vertice* A. *ad ſectionem* D. *producitur, bifariam ſecat trianguli ipſius angulum* A.

a 31. 1. Prob. Ad punctum B. [a] agatur BE. ipſi DA. parallela, b 17. & 29. 1. cui CA. producta [b] occurrat in c 29. 1. E. tunc erit EBA. [c] æqualis alter-

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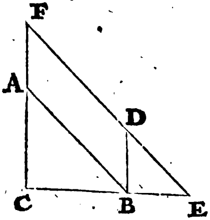
alterno B A D. & E. externo D A C. ergo cum anguli B A D. C A D. æquales ponantur, erunt anguli E B A. & E. æquales, & rectæ B A. A F. [d] æquales. Ergo cum in triangulo E B C. rectæ D A. B E. parallelæ ſint, ut E A. hoc eſt B A. ad A C. [e] ita B D. ad D C. Sit rurſus ut B A. ad A C. ſic B D. ad D C. ut autem B D. ad D C. ita [f] eſt E A. ad A C. [g] Ergo ut B A. ad A C. ita E A. ad A C. [h] æquales ergo B A. E A. & [i] anguli A B E. & E. Cum ergo A B E. alterno B A D. æqualis ſit & E. externo D A C. erunt anguli B A D. D A C. æquales.

d 6. 1.
e 3. 6.
f 26.
g 11. 5.
h 9. 5.
i 5. 1.

PRO-

PROPOSITIO IV.

Th. 4.



Æquiangulorum triangulorum ACB. DBE. *proportionalia ſunt latera* (hoc eſt ut AC. ad CB. ita DB. ad BE) *quæ circa æquales angulos* C. & B. & *homologa ſunt latera* BA. ED. *quæ æqualibus angulis* C. & B. *ſubtenduntur.*

Prob. Sic in directum ſtatue rectas CB. BE. ut angulus extern. DBE. interno C. ſit æqualis: tunc DB. & AC. a erunt parallellæ: ſimiliterque ED. BA. cum anguli E. & ABC. ſint æquales. Et quia anguli ACB. ABC. hoc b eſt DEB. minores ſunt c duobus rectis, ſi producantur ED. CA. convenient d puta in F. e Eritque DA. parallelogrammum. Cum igitur in triangulo FCE. rectæ DB. FB. ſint parallelæ, f erit ut ED. ad DF. hoc eſt BA. ita EB. ad BC. Cumque BA. EF. ſint item parallelæ, erit CB. ad BE. ut CA. ad AF. hoc eſt BD. & ut AB. ad BE. ita ED. hoc eſt AB. ad DE.

a 28. 1.
b 29. 1.
c 17. 1.
d Ax. 11.
e 34. 1.
f 2. 6.

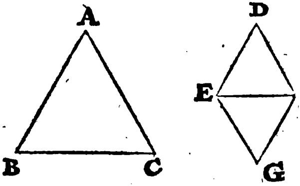
SCHOLIUM.

Quæ hinc vulgo colliguntur nota erunt demonſtrata prop. 8. cum annexo ſcholio.

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PRO-

PROPOSITIO V.



Th. 5.

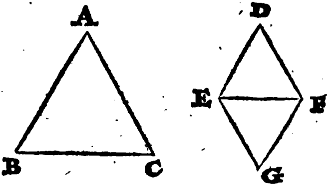
Si duo triangula ABC. DEF. *latera* AB. BC. *proportionalia ipſis* DE. EF. *habuerint, erunt æquiangula, eoſdemque angulos,* DA. EB. FC. *habebunt æquales, quibus homologa latera ſubtenduntur.*

Prob. Super recta EF. ad punctum E. a ponatur angulus FEG. angulo B. æqualis & ad F. alius ipſi C. conſequenter reliquus G. reliquo A. b æqualis, ſicque fiant triangula ABC. EFG. æquiangula; ergo GE. erit ad EF. ut AB. ad BC. hoc eſt ex hypot: DE. ad EF. c quare GE. æqualis erit DE. Simili ratione GF. æqualis eſt DF. cumque latus EF. utrique triangulo commune eſt erunt triangula ABC. & DEF. per. 8. 1. æquiangula &c. Q. E. D.

a 23. 1.
b 32. 1.
c 9. 5.

PROPOSITIO VI.

Th. 6.



Si duo triangula ABC. DEF. *unum habeant æqualem angulum* A. & D. & *latera circa eum proportionalia* (ut BA. ad AC. ita ED. ad DF.) *erunt æquiangula, anguloſque habebunt æquales* E. B. C. F. *quibus homologa latera* BA. ED. AC. DF. *ſubtenduntur*.

Prob. Ad rectam EF. angulos FEG. EFG. fac

æqua-

æquales ipſis B. C. erit & G. æqualis A. quia ergo æquiangula ſunt ABC. GEF. [a] erunt ut AB. ad AC. ita GE. ad GF. proportionalia: ſed ſunt etiam proportionalia AB. AC. & DE. DF. [b] ſunt ergo latera DE. DF. ipſis GE. GF. æqualia. Cumque baſis EF. ſit communis, triangula DEF. EFG. [c] æquiangula ſunt: [d] ergo etiam æquiangula ABC. DEF. Q. E. D.

a 4. 6.

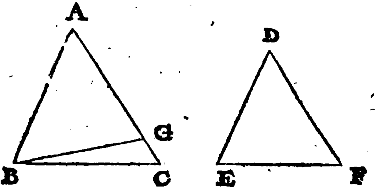
b 11. & 9. 5.

c 8. 1.

d Ax. 1.

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PROPOSITIO VII.



ꝛb. 7. *Si duo triangula* ABC. DEF. *unum angulum* A. *uni angulo* D. *æqualem, circum autem alteros angulos* B. E. *latera proportionalia habeant* (ut AB. ad BC. ita ED. ad EF.) *reliquorum vero* B. E. *ſimul utrumque, aut minorem aut non minorem recto: æquiangula erunt triangula, & æquales habebunt angulos* ABC. DEF. *circũ quos ſunt proportionalia latera.*

Prob. Sit enim C. & F. minor recto, tunc ſi anguli ABC. & E. non ſunt æquales, ſit ABC. major quam E. fiatque ipſi E. æqualis ABG. cum igitur angulus

'3. 1. A. angulo D. ponatur æqualis a

erit

erit & reliquus AGB. reliquo F. æqualis, ideoque triangula ABG. DEF. æquiangula erunt. [b] Ergo ut AB. ad BG. ita erit DE. ad EF. ſed ut DE. ad FE. ita ponitur AB. ad BC. adeoque [c] æquales BG. CB. & [d] anguli BCG. BGC. æquales. Cum igitur angulus C. ſit recto minor erit & BGC. minor recto, & ei deinceps AGB. [e] major recto. Eſt autem oſtenſus angulus AGB. angulo F. æqualis; Major igitur eſt recto angulus F. qui minor ponebatur.

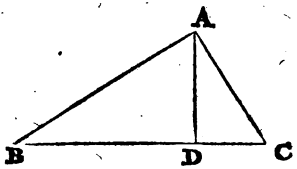
b 4. 6.
c 9. 5.
d 5. 1.
e 13. 1.

Jam ſit angulus B. & E. recto non minor probabitur ut prius rectas BG. & BC. eſſe æquales, & [f] conſequenter angulos BGC. BCG. eſſe æquales, & non minores duobus rectis, [g] quod abſurdum. Non ergo inæquales ſunt anguli ACB. & F. ſed æquales, & conſequenter reliqui anguli B. & E. [h] æquales, quod erat probandum.

f 5. 1.
g 17. 1.
h 32. 1.

PROPOSITIO VIII.

r6. 8.



Si in triangulo rectangulo BAC. *ab angulo recto* A. *in baſim* B C. *perpendicularis* A D. *ducta ſit: quæ ad perpendicularem triangula* ADC. B D A. *tum toti triangulo* BAC. *tum ipſa* A D C. B D A. *inter ſe ſunt ſimilia.*

Prob. In triangulis A B C. D B A. anguli BAC. ADB. recti ſunt & angulus B. communis: ergo [a] reliqui A C B. B A D. æquales: ergo triangula ABC. D B A. [b] ſimilia. Non aliter oſtendetur A B C. ſimile ADC. & A D C. triangulo BDA. Q. E. D.

a 32. 1.

b 1. Def. 6.

Coroll. 1.

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Coroll. 1. Perpendicularis ab angulo recto in baſim, eſt media proportionalis inter duo baſis ſegmenta.

[c] Nam ut BD. ad DA. ita DA. ad DC. quod eſt rectam DA. eſſe mediam proportionalem inter baſis partes BD. DC. c 4. 6.

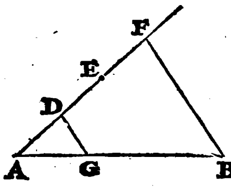
Coroll. 2. Hinc etiam patet utrumlibet laterum rectum ambientium, medium proportionale eſſe inter totam baſim & illud ſegmentum baſis quod ei lateri adjacet.

SCHOLIUM.

Omnes proportiones reſpectu laterum facillimo negotio conſpici poterunt, modo literæ, quibus triangula inſignita ſunt, ordine æqualium angulorum diſponantur & ab utraque parte ſimiliter conferantur, unde etiam corollaria hinc deſumpta patent.

PROPOSITIO IX.

Prob. 1.



A data recta AB. imperatã partẽ puta tertiam AG. auferre.

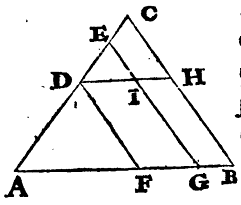
Prax. Ex A. ducatur recta AF. utcunque faciens angulum, & ex AF. ſumatur quævis pars, puta AD. ac duæ aliæ addantur æquales DE. EF. jungatur FB. cui ex D. parallela fiat DG. eritque ablata AG. pars tertia ipſius AB.

Prob. In triangulo AFB. lateri BB. parallela eſt linea GD. [a] ergo erit ut FD. ad DA. ita BG. ad GA. & [b] componendo ut FA. ad DA. ita BA. ad GA. Eſt autem A D. pars tertia ipſius A F. Ergo AG. erit pars tertia ipſius AB. Q. E. F.

a 2. 6.
b 18. 5.

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PROPOSITIO X.



Datam rectam inſectam AB. *ſimiliter ſecare, ut data altera recta* AC. *ſecta fuerit* in D. & E. Prob. 2.

Prax. Jungantur datæ lineæ in A. connectantur recta BC. & ex D. & E. agantur DE. EG. ipſi CB. parallelæ, & factum eſt quod petitur.

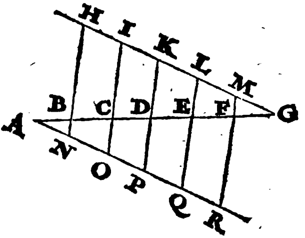
Prob. In triangulo ABC. ductæ ſunt DF. EG. parallelæ lateri BC. a ergo ut AD. ad DE. ita AF. ad FG: Proportionales ergo ſunt partes AF. FG. partibus AD. DE. Jam ſi ducatur DH. parallela ipſi AB. erit ut DE. ad EC. ita DI. ad IH. b hoc eſt FG. ad GB. quare proportionales ſunt partes FG. GB. partibus DE. EC. Q.E.D.

a 2. 6.

b 34. 1.

SCHO-

SCHOLIUM.

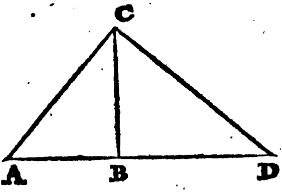


Ex hac & præcedente propoſitione facile conſtat lineam quavis apertura circini in quotvis partes dividere, cujus demonſtrationem & praxin appoſita figura exhibet.

PRO-

PROPOSITIO XI.

Prob. 3.



Datis duabus rectis AB. BC. *tertiam proportionalem invenire.*

Praxis. Duabus datis fac angulum ABC. rectum, item ad AC. angulum rectum ACD. per rectam CD. occurrentem protractæ AB. in D. & factum eſt quod petitur per coroll. 8. cum BC. ſit media proportionalis.

NOTA.

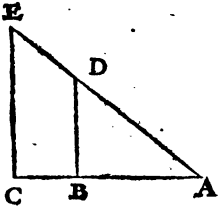
Idem demonſtratur, ut in ſequente, per lineas parallelas; ſumendo tertiam alterutri æqualem.

PRO-

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PROPOSITIO XII.

Prob. 4.



Tribus datis rectis AB. BC. AD. *quartam proportionalem* DE. *invenire.*

Prax. Ex datis, duas AB. BC. in directum colloca, ex reliqua AD. & totali AC. fac angulum DAC. junge recta BD. & fac ipſi parallelam CE. quarta DE. proportionalis erit.

Prob. CE. BD. ſunt parallelæ: [a] ergo ut ſe habet AB. ad BC. ita AD. ad DF. Ergo DE. quarta eſt proportionalis.

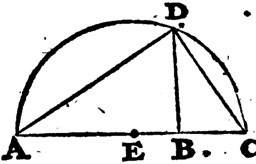
a 2. 6.

NOTA.

Idem conſtat ex 35. prop. lib. 3.

PRO-

PROPOSITIO XIII.



Datis duabus rectis AB. BC. *mediã proportionalem* BD. *Convenire.* Pr. 5.

Prax. Colloca in directum AB. B C. ſuper AC. duc ſemicirculum A D C. In B. excita perpendicularem B D. ad peripheriam ſemicirculi, illa erit quæſita.

Prob. Ductis rectis AD. CD. a erit angulus A D C. in ſemicirculo rectus, & à vertice D. ad baſim A C. ducta perpendicularis D B. facit b duo triangula æquiangula: c ergo proportionalia: ergo ut AB. ad BD. ita BD. ad B C. eſt ergo BD. media proportionalis inter AB. BC. Q.E.D.

a 31. 3.
b 8. 6.
c 4. 6.

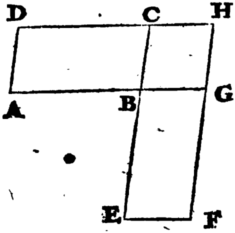
Corollarium.

Hinc quævis recta à circumferentia ad diametrum perpendicularis ducta, media proportionalis eſt inter diametri ſegmenta.

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PROPOSITIO XIV.

Th. 9.



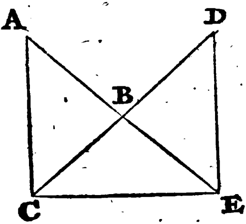
Æqualium A C. B F. *& unum* ABC. *uni* EBG. *æqualem habentiũ angulum, parallelogrammorum, reciproca ſunt latera* A B. B G. E B. B C. *quæ circum æquales angulos: & quorum parallelogrammorum, unum angulum uni angulo, æqualem habentiũ, reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.*

Prob. Jungantur parallelogramma ad angulum æqualem B. ita ut A B.
a 14. & B G. jaceant in directum a jace-
& 15.1. bunt & reliquæ E B. B C. perficiatur parallelogrammũ BH. ergo ut FB ad BH ita
b 7.5. b erit BD. ad BH. ſed ut FB. ad BH. ita c eſt
c 1.6. EB. ad BC. & ut DB. ad BH ita AB. ad BG.
d 11.5. igitur ut E B. ad BC. d ita eſt AB. ad BG.

Prob. 2. pars. Ex hypoth. EB. ad B C.
e 1.6. eſt ut AB. ad B G. ergo e E G. ad BH. eſt
f 9.5. ut DB. ad B H. f ergo parallelogramma æqualia ſunt. Q. E. D.

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PROPOSITIO XV.



Th. 10.

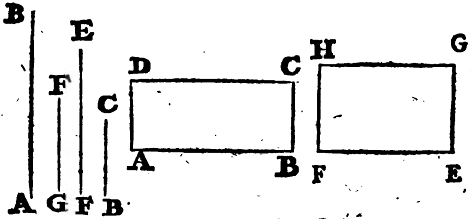
Æqualium A B C. D B. E. *& unum* B. *uni* B. *æqualem habentium angulum, triangulorum reciproca ſnnt latera* ut AB. ad BE. ita DB. ad BC. *quæ circum æquales angulos* B. *& quorum triangulorum, unum angulum uni æqualem habentium reciproca ſunt latera, quæ circum æquales angulos, illa ſunt æqualia.*

Prob. Sic junge triangula ad angulum æqualem B. ut AB. BE. jaceant in directum, ducta CE. a erit ut A B C. ad B C E. ita DBE. ad B C E. ſed ut ABC. ad B C E. ita b A B. ad BE. & b ut D B E. ad B C E. ita B D. ad BC. Similiter demonſtratur A B C. D B E. eſſe æqualia, ſi ſit ut AB. ad BE. ita DB. ad B C. Nam cum ponatur ut AB. ad B E. ita D B. ad B C. & ut A B. ad B E. ita triangulum ABC. ad B C E. & ut D B. ad B C. ita DBE. ad BCE. erit ut A B C. ad BCE. ita DBE. ad B C E. ergo triangula ABC. DBE. c ſunt æqualia. Q.E.D.

a 7. 5.
b 1. 6.
c 9. 5.

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PROPOSITIO XVI.



Th. 11. *Si quatuor rectæ* A.G.F.B. *proportionales fuerint: quod ſub extremis* AB. BC. *comprehenditur rectangulum* AC. *æquale eſt ei, quod ſub mediis* EF. FG. *comprehenditur rectangulo* FG. *Et ſi ſub extremis* AB. BC. *comprehenſum rectangulum* AC. *æquale fuerit ei quod ſub mediis* GF. FE. *continetur rectangulo* FG. *illa quatuor rectæ proportionales ſunt.*

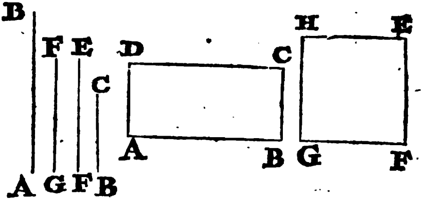
Prob. 1. Anguli recti B. & E. ſunt æquales, & ut ſe habet AB. ad EG. ita EF ad BC. ergo latera circa æquales angulos B.& E. ſunt reciproca, a ergo parallelogramma AC. FG. ſunt æqualia. (a 14. 6.)

Prob 2. Æqualia ſunt rectangula AC. FG. & habent angulos æquales, nempe rectos B. & E. ergo b latera circa hos angulos reciproca erunt etiam b proportionalia. Q. E. D. (b 14. 6.)

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PRO-

PROPOSITIO XVII.



Si tres rectæ A F B. *ſint proportionales: quod ſub extremis* A B. B C. *comprehenditur rectangulum* A C. *æquale eſt ei, quod à media* F. *deſcribitur quadrato* E G. *Et ſi ſub extremis* A B. A C. *comprehenſum rectangulum* A C. *æquale ſit ei quod à media* F. *deſcribitur quadrato* E G. *illæ tres rectæ proportionales erunt.* Th. 12.

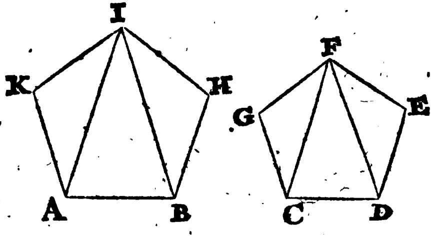
Prob. 1. pars. Sume rectam G F. æqualem ipſi F E. erunt quatuor rectæ A G. F B. proportionales, eritque quadratum E G. comprehenſum ſub mediis F G. E F. a ergo rectangulum A C. æquale erit quadrato G E.

Prob. 2. Quadratum F G. mediæ E F. (vocemus parallelogrammum) rectangulo A C. ſub extremis A B. B C. æquale ponitur, & habent angulos æquales: ergo latera ut proxime dixi, circa hos angulos erunt reciproca adeoque proportionalia. a 16. 6.

N O T A.

Ex hac & præcedenti, cujus quaſi repetitio eſt, infertur fundamentum regulæ vulgò dictæ de Tri, &c.

PROPOSITIO XVIII.



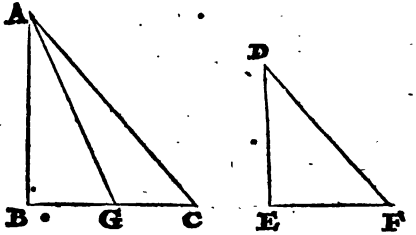
Prob. 6. *Super data recta* AB. *dato rectilineo* CDEFG. *ſimile, ſimiliterque poſitum rectilineum* ABHIK. *deſcribere.*

Datum rectilineum reſolve in triangula, ductis rectis puta CF. DF. Ad punctum
a 32. 1. A. a fiat angulus IAB. æqualis ipſi FCD. & ipſi FDC.
b 32. 1. æqualis IBA. & b conſequenter reliquus reliquo: Æquiangula ergo erunt triangula FCD. IAB.
c 4. 6. & ſimilia c & ut CF. ad AI. ita CD. ad AB. Ad rectam AI. fac ſimi-

ſimiliter triangulum IKA. æquiangulum triangulo FGC. & quia anguli BAF. IAK. æquales ſunt angulis DCF. FCG. totales KAB. GCD. æquales erunt, & latera proportionalia. Idemque repetendum, donec omnia triangula eodem ordine quo jacent abſolvantur, ſicque totum rectilineum toti rectilineo [d] ſimile erit, & ſuper datam AB. ſimiliter deſcriptum. Q. E. F.

[d] 1. Def.

PROPOSITIO XIX.



Th. 13. *Similia triangula* ABC. DEF. *inter ſe ſunt in duplicata ratione laterum homologorum.*

Quando triangula ſunt æqualia, hoc eſt quando B C. E F. nec non tertia proportionalis B G. ſunt æquales, res eſt manifeſta.

Quando vero latera B C. E F. ſunt inæqualia, demonſtratur, hoc modo. Sit B C. latus, latere E F. majus, & ex B C.
a 11. 6. abſcindatur [a] rectis B C. E F. tertià proportionalis B G. ducaturque recta A G. Quia igitur angu-

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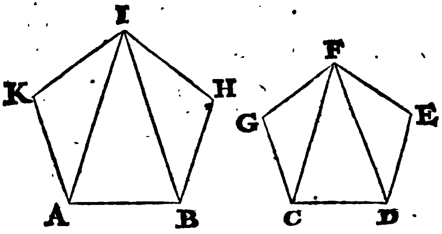
angulus B. eſt æqualis E. & propter ſimilitudinem triangulorum, ut A B. ad B C. ita D E. ad E F. & permutando ut A B. ad D E. ita B C. ad E F. hoc eſt E F. ad B G. erunt circa angulos æquales B. E. latera reciprocè proportionalia. Quare per 14. triangula A B G. D E F. erunt æqualia; & per 7. quinti ut triangulum A B C. ad A B G. ita erit idem triangulum A B C. ad D E F. ut autem A B C. ad A B G. ita eſt per 1. hujus B C. ad B G. Ergo A B C. ad DEF. erit ut B C. ad B G. hoc eſt in duplicata ratione per 10. def. 5. Q. E. D.

Corollarium.

Si tres lineæ fuerint proportionales, ut prima ad tertiam, ita triangulum ſuper primam ad ſimile triangulum ſuper ſecundam.

PRO-

PROPOSITIO XX.



Th. 14. *Similia poligona in ſimilia triangula dividuntur, & numero æqualia, & totis homologa: & polygona duplicatam habent eam inter ſe rationem, quam latus homologum ad homologum latus.*

Sint polygona ſimilia ABHIK. CDEFG. habentia angulos æquales K. G. Itemque I. F. & ſic deinceps, & latera proportionalia circa angulos æquales, puta ut AB. ad BH. ita CD. ad DE. &c.

Dico 1. illa dividi in triangula ſimilia & numero æqualia. Prob. ab angulis I. & F. duc rectas ad angulos oppoſitos AB. CD. diviſa erunt illa polygona in triangula numero æqualia: quod etiam ſimilia ſunt.

Prob. Anguli K. & G. ſunt æquales, & a 6. 6. circa ipſos latera ſunt proportionalia. a ergo æquiangula ſunt triangula IKA. FGC. ergo ſimilia. Eadem ratione erunt ſimilia triangula IHB. FED. dein ut IB. ad BH. ita FD. ad DE. ut autem HB. ad BA. ita b 22. 5. ED. ponitur ad DC. b erit ex æquo ut IB. ad BA.

ad BA. ita FD. ad DC. & quoniam angulus HBA. ipſi EDC. eſt æqualis, & ablatus HBI. ablato EDF. erunt reliqui IBA. FDC. æquales. c Ergo triangula IBA. FDC. æquiangula erunt & ſimilia, eademque ratio de omnibus. c. 6. 6.

Dico 2. quod ſicut unum triangulum ad triangulum ſibi reſpondens alterius polygoni: ita eſſe polygona tota inter ſe.

Prob. Quia omnia triangula ſunt ſimilia, ſingula ſingulis: d ergo ſunt in duplicata ratione laterum homologorum; cumque ſingula ſingulis probata ſint proportionalia, ſic ut in triangulo unius ſint omnia antecedentia, in alio conſequentia proportionum, e ut unum antecedens eſt ad unum conſequens ita omnia antecedentia ad omnia conſequentia. Eſt ergo polygonum ad polygonum ut triangulum ad triangulum: ergo ea triangula ſunt totis homologa, & quia triangula ſunt in duplicata ratione laterum homologorum, erunt & polygona in eadem ratione duplicata laterum homologorum puta AB. CD. Q. E. D. d 19. 6. e 12. 5.

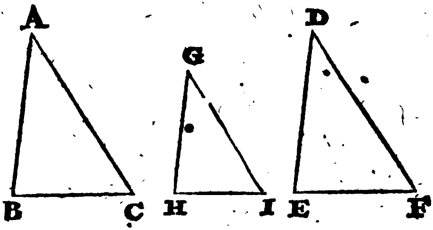
Corollarium.

Hinc ſi fuerint tres rectæ proportionales, ut eſt prima ad tertiam ita polygonum ſuper primam deſcriptum ad polygonum ſuper ſecundam ſimile ſimiliterque deſcriptum, vel etiam polygonum ſuper tertiam ſimile ſimiliterque deſcriptum.

PRO-

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PROPOSITIO XXI.



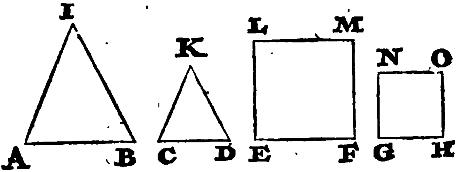
Th. 15. *Quæ eidem rectilineo* GHI. *ſunt ſimilia* ABC. DEF. *& inter ſe ſunt ſimilia.*

Prob. Anguli A. & D. ponuntur æquales uni G. ergo & inter ſe, eodemque modo ſinguli ſingulis: [a] latera etiam circa eos ponuntur proportionalia, quia lateribus ejuſdem tertii ſunt proportionalia: ergo cum habeant angulos æquales & latera circa eos proportionalia, [b] ſunt ſimilia. Q. E. D.

a 11. 5.
b 1. Def. 6.

PRO-

PROPOSITIO XXII.



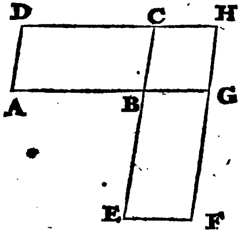
Si quatuor rectæ AB. CD. EF. GH. *proportionales fuerint: & ab eis rectilinea ſimilia ſimiliterque deſcripta* ABI. CDC. & MF. NH. *proportionalia erunt. Et ſi à rectis lineis, ſimilia, ſimiliterque deſcripta rectilinea proportionalia fuerint, ipſæ rectæ proportionales erunt.* Tb. 16.

Prob. Triangulum ABI. eſt ad triangulum CDK. in duplicata a ratione lateris AB. ad CD. ſimiliter EM. ad GO. ut EF. ad GH. adeoque erit ABI. ad CDK. ut EM. ad GO. Q. E. D. Jam vero ſi figuræ proportionales & ſimiles ſimiliterque poſitæ ſint, & rectæ ſuper quas poſitæ ſunt, proportionales erunt: nam ratio unius figuræ ad alteram b eſt rectæ ad rectam duplicata: c ergo ratio laterum eadem erit, nempe ut AB. ad CD. ita EF. ad GH. ergo illarum latera proportionalia erunt. Q. E. D.

a 19. 6.
b 19. & 20. 6.
c 7. 5.

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PROPOSITIO XXIII.



Th. 17. *Æquiangula parallelogramma* A C. B F. *inter ſe rationem habent eam, qua ex lateribus componitur* A B. *ad* B G. *&* E B. *ad* B C.

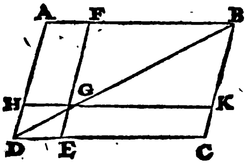
Sint parallelogramma AC. BF. habentia angulos ad B. æquales, & ita diſpoſita ut appoſita figura reſultet. Nunc ratio A C. ad B F. æqualis eſt rationi [a]. A C. ad B H. una cum ratione B H. ad B F. itidem æqualis rationi [b]. AB. ad BG. cum ratione CB. ad BE. Q. E. D.

a 20. Def. 5.

b 1. 6.

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PROPOSITIO XXIV.

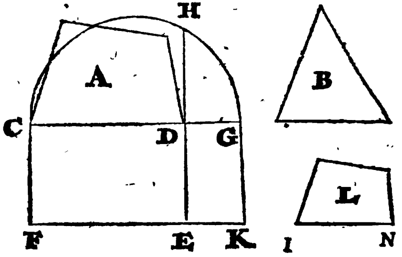


In omni parallelogrammo A C. *quæ circa diametrum* D B. *ſunt parallelogramma* FK. HE. *& toti* A C. *& inter ſe ſunt ſimilia.* *Th.* 18.

Parallelogramma H E. F K. cum toto angulum communem habentia reliquoſque per 29. 1. æquales ut BAD. GHD. BFG. ipſis B C D. G E D. B K G. æquiangula erunt, adeoque latera per 4. 6. proportionalia, conſtituunt parallelogramma cum toto & inter ſe ſimilia. Q. E. D.

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PROPOSITIO XXV.

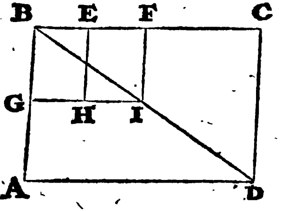


Prob. 7. *Dato rectilineo* A. *ſimile, ſimiliterque poſitum, & alteri dato* B. *æquale* L. *conſtituere.*

Prax. Ad dati rectilinei A. latus CD.
a fiat rectangulum CE. æquale ipſi
a 45. 1. A. Producatur CD. verſus G. ſuper
DE. in angulo EDG. fiat rectangulum
b 44. 1. DK. b æquale ipſi B. c fiat inter CD. DG.
c 13. 6. media proportionalis DH. æqualis ipſi IN.
d 18. 6. ſuper quam fiat d rectilineũ L. ſimile ipſi
A. ſimiliterque poſitum, eritque rectilineum L. æquale dato B. & ſimile ipſi A.
e *Ex conſt.* Prob. Rectæ CD. DH ſeu IN. DG. e ſunt
f 19. & proportionales: f ergo erit ut prima CD.
20. 6. ad tertiam DG. ita rectilineum ſuper
primam, id eſt A. ad rectilineum ſuper
g 1. 6. ſecundam, id eſt L. ſed ut CD. ad DG. g ita
parallelogrammum CE. hoc eſt A. ad
h 12. 5. DK. hoc eſt B. h ergo erit ut A. ad B. ita
i 9. 5. A. ad L. i ideoque rectilinea B. & L.
erunt æqualia. Q. E. D.

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PROPOSITIO XXVI.



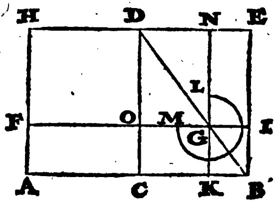
Si à parallelogrammo BD. *parallelogrammum* FG. *ablatum ſit, & ſimile toti, & ſimiliter poſitum: communem cum eo habens angulum* FBG. *circa eandem cum toto diametrum* BD. *conſiſtet.* Th. 19.

Si neges: tranſeat alibi diameter puta per H. à quo puncto ducatur ex H. recta HE. parallela BG. tunc parallelogramma BD. BH. circa eandem diametrum BHD. a erunt ſimilia: b quare erit ut BA. ad AD. ita BG. ad GH. Sed ut BA. ad AD. ita BG. ad GI. unde per 9. 5. GH. æqualis GI. pars toti. Q.E.A.

a 24. 6.
b

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PROPOSITIO XXVII.



6. 20. *Omnium parallelogrammorum ſecundum eandem rectam applicatorum deficientiumque figuris parallelogrammis ſimilibus, ſimiliterque poſitis, ei quod à dimidia deſcribitur; maximum eſt id quod ad dimidiam applicatur parallelogrammum ſimile exiſtens defectui.*

SUPER AC. ſemiſſem totius AB. applicatum ſit parallelogrammum AD. ita ut à toto AE. deficiat parallelogrammo CE. quod eſt æquale & ſimile ipſi AD. Deinde ad quodvis aliud ſegmentum AK. ſit appli-

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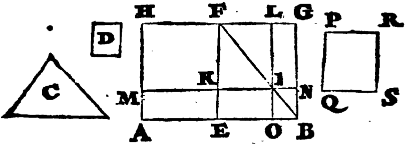
catum aliud parallelogrammum A G. ita deficiens, ut defectus ſit parallelogrammum K I. ſimile ipſi C E. hoc eſt circa communem diametrum B G D. Dico A G. minus eſſe parallelogrammo A D. Probatur.

1. Parallelogramma A D. C E. F D. O E. ſunt [a] æqualia ut & [b] C G. G E. adeoque addito communi K I. erit C I. hoc eſt A O. æquale ipſi K E. addito communi C G. erit A G. æqualis gnomoni L G M. minor parall. C E. hoc eſt A D. parall. Q. E. D.

a 36. 1.
b 43. 1.

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PROPOSITIO XXVIII.



Prob. 8. *Ad datam rectam* AB. *dato rectilineo* C. *æquale parallelogrammum* AI. *applicare: deficiens figura parallelogramma* ON. *quæ ſimilis ſit alteri parallelogrammo dato* D. *Oportet autem datum rectilineum* C. *cui æquale applicandum eſt* A I. *non majus eſſe eo, quod ad dimidiam* A E. *applicatur, cum ſimiles fuerint defectus, & ejus quod ad dimidiam applicatur, & ejus cui ſimile deeſſe debet.*

Rectam A B. ut prius biſeca in E. ſuper mediam E B. fac parallelogrammum E G. ſimile ipſi D. ſimiliterque poſitum: & comple parallelogrammum B H. Si E H. ipſi C. eſt æquale, factum eſt quod petitur: nam eſt applicatum ad A B. & deficit parallelogrammo E G. ſimili ipſi D. Si E H.

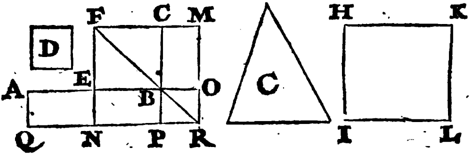
& ipſi

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& ipſi æquale b EG. ſit majus quam C. b 36. 1.
(nam minus eſſe non debet, cum EH.
ſit c maximum eorum quæ applicari c 27. 6.
poſſunt ad AB.) ſi inquam ſit majus, d 45. 1.
d reperta quantitate exceſſus, e fac pa- aut arte
callelogramm um QR æquale exceſ- quacunque.
ſui, & ſimile ſimiliterque poſitum ipſi
D. & parallelogrammo QR. aliud æ- e 25. 6.
quale ſimiliter poſitum KL. f quod f 44. 1.
erit circa diametrum, ſicque remanebit gnomon LIK. æquale rectilineo C. Jam productis LI. KI. erit parallelogrammum AI. ad rectam AB. applicatum & deficiens parallelogrammo
ON. g ſimili ipſi EG. hoc eſt ipſi D. g 24. 6.
Quod autem AI. ſit æquale ipſi C. ſic probo. Complementa LN. KO.
h ſunt æqualia, ergo addito communi g 43. 1.
NO. erit OG. æquale ipſi EN. hoc eſt AK. Ergo ſi æqualibus AK. OG. addas commune KO. erit AI. æquale gnomoni LIK. hoc eſt rectilineo C. Q. E. F.

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PROPOSITIO XXIX.



Prob. 9. *Ad datam rectam* AB. *dato rectilineo* C. *æquale parallelogrammum applicare, excedens rectam datam* AB. *figura parallelogramma* PO. *quæ ſit ſimilis dato alteri parallelogrammo* D.

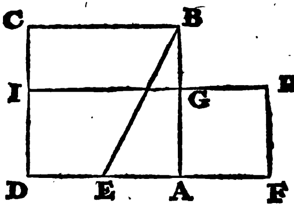
a 18.6. Super rectam EB. mediam datæ AB. [a] fiat parallelogrammum EC. ſimile ipſi D. ſimiliterque poſitum: tum rectilineo C. & parallelogrammo

b 25.6. EC. fiat [b] æquale aliud parallelogrammum IK. cui æquale eſt NM. ſimile ipſi D. Completis parallelogrammis QE. NB. PO.

PO. erit AR. quæſitum. Etenim NM. eſt poſitum æquale ipſis EC. & C, ablato communi EC. gnomon ERC. ipſi C. erit æqualis. Et quia æqualia [c] c 36.1.
ſunt QE. NB. & æqualia [d] NB. BM. ſi loco ipſius d 43.1.
BM. ſubſtituatur æquale QE. erit parallelogrammum AR. æquale gnomoni ERC. ideoque etiam rectilineo C. Quare ad rectam AB. applicatum eſt parallelogrammum AR. æquale dato rectilineo C. excedens rectam AB. figura parallelogramma PO. quæ ſimilis eſt dato parallelogrammo D. cum ſit circa eandem diametrum cum ipſo EC. quod poſitum eſt ſimile ipſi D. Q.E.F.

PRO-

PROPOSITIO XXX.



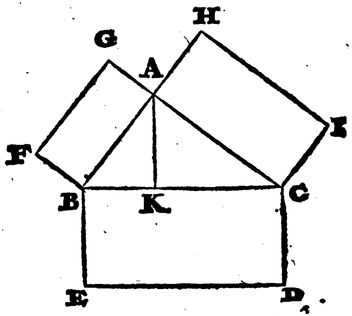
Pr. 10. *Propoſitam rectam terminatam AB. extrema ac media ratione ſecare in G.*

a 11.2. [a] DIVIDATUR AB. in G. ita ut rectangulum CG. ſub tota AB. & ſegmento BG. ſit æquale quadrato AH. alterius b 17.6. ſegmenti AG. tunc enim tres rectæ proportionales [b] erunt; & erit ut tota AB. ad AG. ita c 3.Def. AG. ad GB. Ergo AB. ſecta eſt in G. [c] ſecundum extremam, & mediam rationem. Q. E. F.

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PROPOSITIO XXXI.



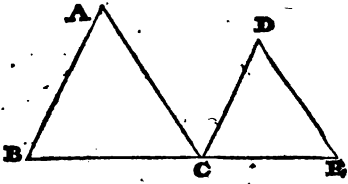
In triangulo rectangulo ABC. *figura quævis* BD. *deſcripta à* BC. *ſubtendente rectum angulum* BAC. *æqualis eſt figuris* FA. AI. *quæ priori illi ſimiles & ſimiliter poſitæ, à lateribus* BA. CA. *rectum angulum continentibus, deſcribuntur.* Th. 26.

POLYGONÆ figuræ FA. AI. BD. ponuntur ſimiles a ergo ſunt in ea laterum homologorum duplicata ratione, in qua eſſent eorundem laterum quadrata. Ergo cum quadrata BA. AC. b habeant rationem æqualitatis cum tertio BC. habebunt & polygona FA. AI. rationem æqualitatis cum tertio BD. c ergo eidem erunt æqualia. Q.E.D.

a 20. 6.
b 47. 1.
c 9. 5.

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PROPOSITIO XXXII.



Th. 21. *Si duo triangula* ABC. DCE. *quæ duo latera* AB. AC. *duobus lateribus* DC. DE. *proportionalia habeant, ſecundum unum angulum* ACD. *compoſita fuerint, ita ut homologa eorum latera* AB. DC. AC. DE. *ſint etiam parellela, tum reliqua illorum triangulorum latera* BC. CE. *in rectam lineam* BE. *collocata reperientur.*

PROB. Latera homologa AB. DC. AC. DE. ponuntur
a 29.1. parallela, [a] ergo anguli alterni A. & ACD. ſunt æquales & D. eidem ACD. ergo A. & D.

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æqua-

æquales. Hos æquales angulos circumſtant latera proportionalia ex hypoth. [b] ergo triangula ſunt æquiangula, habentque æquales angulos B. & DCE. additis ergo æqualibus A. & ACD. erunt B. & A. duobus angulis DCE. ACD. hoc eſt angulo ACE. æquales. Ergo addito communi ACB. erunt tres anguli A.B.C. duobus ACE. ACB. æquales, [c] illi autem tres valent duos rectos, ergo & hi duo. Ergo [d] BC, CE. unam rectam conſtituunt. Q. E. D.

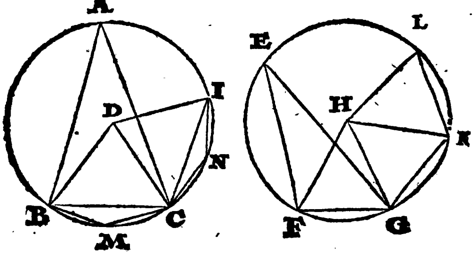
b 6. 6.

c 32. 1.

d 14. 1.

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PROPOSITIO XXXIII.



Th. 22. *In æqualibus circulis* ABCI. EFGL. *anguli* A. E. D. H. *eandem habent rationem, cum ipſis peripheriis* BC FG. *quibus inſiſtunt: ſive ad centra* D. H. *ſive ad peripherias* A. E. *conſtituti inſiſtant: inſuper vero & ſectores* BDC. FHG. *quippe qui ad centra, inſiſtunt.*

a 1.4. PROB. Ductis B C. F G. ad C. [a] applica CI. æqualem ipſi B C. & ad G. & K. GK. KL. æquales ſingulas ipſi FG.

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FG. ductis I D. K H. L H. ſic dico; rectæ B C. C I. ponuntur æquales, [b] ergo & arcus B C. C I. [c] ergo & anguli BDC. CDI. æquales. Idemque eſt de arcubus F G. G K. K L. & angulis ad H. qui ipſis inſiſtunt. Ergo quam multiplex eſt arcus B C I. ipſius B C. tam multiplex erit angulus B D I. ipſius B D C. & quam multiplex arcus F G K L. ipſius F G. tam multiplex erit angulus F H L. ipſius F H G. [d] ergo ſi arcus B C I. F G K L. ſint æquales, erunt & anguli B D I. F H L. æquales. Si eorum arcuum unus ſit major, major erit & angulus, ſi minor, minor: [e] Ergo erit ratio arcus B C. ad F G. eadem quæ eſt anguli B D C. ad FHG. Et quia anguli ad D. & H. ſunt [f] dupli angulorum ad A. & E. [g] eadem erit ratio angulorum A. & E. quæ D. ad H. & ſic eadem

b 28. 3.
c 27. 3.
d 27. 3
e 6. Def. 5.
f 20. 3.
g 15. 5.

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anguli A. ad angulum E. quæ arcus BC. ad arcum FG.

Rurſus, in æqualibus ſegmentis BC. CI. ſi fiant anguli B M C
h 27. 3. C N I. [h] æquales erunt, cum inſiſtant æqualibus arcubus B A C.
i 24. 3. C A I. ergo [i] ſimilia ſunt ſegmenta B M C. C N I. & æqualia, cun ſunt ſuper æquales B C. C I. additis ergo triangulis BDC. CD quæ æqualia ſunt, erunt ſectore BDC. CDI. æquales. Ergo tai multiplex eſt ſector B D I. ſecto ris B D C. quam multiplex arcu B C I. arcus B M C. Idem oſten detur de ſectore FHL. Ergo ſi a qualis ſit arcus B C I. arcui FGI ſector quoque B D I. æqualis er ſectori FHL. ſi deficiat, deficie ſi excedat, excedet. Ergo quæ e ratio arcus B C. ad arcum F G eadem erit & ſectoris B D C. a ſectorem F H G. Q. E. D.

Selectiores hujus libri ſunt 1. 2. 3. 5. 6. 8. 13. 14. 16. 19. 31.

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